Rationality in the Behaviour of Slime Moulds and the Individual-Collective Duality

Andrew Schumann[1]

Abstract. We introduce the notion of the so-called context-based games to describe rationality of the slime mould. In these games we assume that, first, strategies can change permanently, second, players cannot be defined as individuals performing just one action at each time step. They can perform many actions simultaneously. In other words, each player can behave as an individual or as a collective of individuals. This significant feature of context-based games is called individual-collective duality.

1 INTRODUCTION

In *Physarum Chip Project: Growing Computers From Slime Mould* [1] supported by FP7 we are going to design an unconventional computer on programmable behaviour of *Physarum polycephalum*, a one-cell organism that behaves by its plasmodium that is sensible to different stimuli called attractants, it looks for them and in case it finds them, it propagates protoplasmic tubes toward those attractants. These motions can be regarded as the basic medium of simple actions that are intelligent [1], [3], [4].

Notice that the *Physarum* motions are a kind of *natural transition systems*, ⟨States, Edg⟩, where States is a set of states presented by attractants and Edg ⊆ States × States is a transition of plasmodium from one attractant to another. The point is that the plasmodium looks for attractants, propagates protoplasmic tubes towards them, feeds on them and goes on. As a result, a transition system is built up. Now, labelled transition systems have been used for defining the so-called *concurrent games*, a new semantics for games proposed by Samson Abramsky. Traditionally, a play of the game is formalized as a sequence of moves. This way assumes the polarization of two-person games, when in each position there is only one player's turn to move. In concurrent games, players can move concurrently.

On the medium of *Physarum polycephalum* we can, first, define concurrent games and, second, extend the notion of concurrent games strongly and introduce the so-called *context-based games*. In these games we assume that strategies can change permanently. Another feature of context-based games is that players cannot be defined as individuals who perform just one action at each time step. They can perform many actions simultaneously. So, each player can behave as an individual or as a collective of individuals. This significant feature of context-based games is called *individual-collective duality*.

In this paper we will talk about the notion of rationality within context-based games.

2 ACTIONS OF PLASMODIA

Physarum polycephalum verifies the following three basic operations which transform one states to others in ⟨States, Edg⟩: fusion, multiplication, and direction. (i) The *fusion* means that two active zones (attractants occupied by the plasmodium) either produce new active zone (i.e. there is a collision of the active zones) or just a protoplasmic tube. (ii) The *multiplication* means that the active zone splits into two independent active zones propagating along their own trajectories. (iii) The *direction* means that the active zone is not translated to a source of nutrients but to a domain of an active space with certain initial velocity vector. These three operations can be examined as the most basic forms of intelligent behaviour of living organisms. For example, in the paper [4] we showed that the behaviour of collectives of the genus Trichobilharzia Skrjabin & Zakharov, 1920 (Schistosomatidae Stiles & Hassall, 1898) can be simulated in the *Physarum* spatial logic. This means that, first, a local group of Schistosomatidae can behave as a programmable biological computer, second, a biologized kind of process calculus such as *Physarum* transition system can describe concurrent biological processes at all.

The main result of our research is that, on the one hand, the Physarum motions are intelligent, but, on the other hand, they do not verify the *induction principle* (when the minimal set satisfying appropriate properties is given). This means that they can implement Kolmogorov-Uspensky machines or other spatial algorithms only in a form of approximation, because Physarum performs much more, than just conventional calculations (the set realised is not minimal), i.e. it achieves goals (attractants) not only by “Caesarian” straight paths.

Let us consider the following thought experiment as counterexample showing that the set of actions for the plasmodium is infinite in principle, therefore we cannot implement Kolmogorov-Uspensky machines. Assume that the transition system for the plasmodium consists just of one action presented by one neighbour attractant. The plasmodium is expected to propagate a protoplasmic tube towards this attractant. Now, let us place a barrier with one slit in front of the plasmodium. Because of this slit, the plasmodium can be propagated according to the shortest distance between two points and in this case the plasmodium does not pay attention on the barrier. However, sometimes the plasmodium can evaluate the same barrier as a repellent for any case and it gets round the barrier to reach the attractant according to the longest distance. So, even if the environment conditions change a little bit, the

[1] Dept. of Social Science, Univ. of Information Technology and Management in Rzeszow, Sucharskiego 2, 35-225 Rzeszow, Poland. Email: andrew.schumann@gmail.com.

behaviour changes, too. The plasmodium is very sensible to the environment.

Thus, simple actions of Physarum plasmodia cannot be regarded as atomic so that composite actions can be obtained over them inductively. In other words, it is ever possible to face a hybrid action which is singular, but it is not one of the basic simple actions. It is a hybrid of them.

In the transition system with only one stimulus presented by one attractant, a passable barrier can be evaluated as a repellent 'for any case'. Therefore the transition system with only one stimulus and one passable barrier may have the following three simple actions: (i) pass trough, (ii) avoid from left, (iii) avoid from right. But in essence, we deal only with one stimulus and, therefore, with one action, although this action has the three modifications defined above.

Simple actions which have modifications depending on the environment are called *hybrid*. The problem is that the set of actions in any labelled transition systems must consist of the so-called atomic actions – simple actions that have no modifications.

3 INDIVIDUAL-COLLECTIVE DUALITY AND NON-ADDITIVITY

In context-based games, we cannot use conventional probability theory. The matter is that if we assume the existence of hybrid actions, then the entities of games are certain and, therefore, cannot be additive.

The double slit experiment with the plasmodium of *Physarum polycephalum* is the best example of that conventional probability theory is unapplied for *Physarum* acts. Let us take the first screen with two slits which are covered or opened and the second screen behind the first at which attractants are distributed evenly. Before the first screen there is an active zone of plasmodium. Then let us perform the following three experiments: (i) slit 1 is opened, slit 2 is covered; (ii) slit 1 is covered, slit 2 is opened; (iii) both slit 1 and 2 are opened. In the first (second) experiment protoplasmic tubes arrive at the screen at random in a region somewhere opposite the position of slit 1 (slit 2). Let us denote all tubes landing at the second screen by A, thereby all tubes that pass through slit 1 by A_1 and all tubes that pass through slit 2 by A_2. Now we can check in case of *Physarum* if there is a partition of set A into sets A_1 and A_2. We open both slits. Then we see that the plasmodium behaves like electrons, namely it can propagates just one tube passing through either slit 1 or slit 2 or it can propagates two tubes passing through both slits simultaneously. In the second case, these tubes split before the second screen and appear to occur randomly across the whole screen. Thus, the total probability P(A), corresponding to the intensity of plasmodium reaching the screen, is not just the sum of the probabilities $P(A_1)$ and $P(A_2)$. This means that the plasmodium has the fundamental property of electrons, discovered in the double-slit experiment. It is the proof of non-additivity of probabilities.

Economics and conventional business intelligence tries to continue the empiricist tradition, where reality is measurable and additive, and in statistical and econometric tools they deal only with the measurable additive aspects of reality. They try to obtain additive measures in economics and studies of real intelligent behaviour, also. Nevertheless, there is always the possibility that there are important variables of economic systems which are unobservable and non-additive in principle. We should understand that statistical and econometric methods can be rigorously applied in economics just after the presupposition that the phenomena of our social world are ruled by stable causal relations between variables. However, let us assume that we have obtained a fixed parameter model with values estimated in specific spatio-temporal contexts. Can it be exportable to totally different contexts? Are real social systems governed by stable causal mechanisms with atomistic and additive features?

Hence, our study of context-based games on the medium of *Physarum polycephalum* can make impacts for many behavioural sciences: game theory, behavioural economics, behavioural finance, etc.

Non-additivity of phenomena does not mean that they cannot be studied mathematically. There are some rigorous approaches such as p-adic probability theory, which allow us to do it. The most significant feature of p-adic probabilities (or more generally, non-Archimedean probabilities or probabilities on infinite streams) is that they do not satisfy additivity. On the one hand, the p-adic analogies of the central limit theorem in real numbers face the problem that the normalized sums of independent and i.d. random variables do not converge to a unique distribution, there are many limit points, therefore there is no connection with the usual bell type curve. In other words, in p-adic distributions we cannot build up the Gauss curve as fundamental notion of statistics and econometrics. On the other hand, the powerset over infinite streams like p-adic numbers is not a Boolean algebra in general case. In particular, there is no additivity (we cannot obtain a partition for any set into disjoint subsets whose sum gives the whole set). Using p-adic (non-Archimedean) probabilities we can disprove Aumann's agreement theorem and develop new mathematical tools for game theory, in particular define context-based games by means of coalgebras or cellular automata. In these context-based games we can appeal just to non-Archimedean probabilities. These games can describe and formalize complex reflexive processes of behavioural finances (such as short selling or long buying).

Notice that the p-adic number system for any prime number p extends the ordinary arithmetic of the rational numbers in a way different from the extension of the rational number system to the real and complex number systems. The extension is achieved by an alternative interpretation of the concept of absolute value.

Let us suppose that the sample space of probability theory is not fixed, but changes continuously. It can grow, be expanded, decrease or just change in itself. In this case we will deal not with atoms as members of sample space, but with streams. The powerset of this growing set cannot be a Boolean algebra and probability measure is not additive.

We can consider *Physarum* behaviours within a certain topology of attractants and repellents as growing sample space. Assume that there are two neighbour attractants a and b. We say that there is a string ab or ba if both attractants a and b are occupied by the plasmodium. As a result, we observe a continuous expansion of the set of strings. It can be regarded as a sample space of probability theory. Its values will be presented by p-adic integers.

Let us show, how we can build up the sample space Ω^{ω} constructively. Suppose that Ω consists of $p - 1$ attractants and A, B, ... are subsets of Ω. Such A, B, ... are conditions (properties) of the experiment we are performing. For instance, let A := "Attractants accessible for the attractant N_1 by protoplasmic tubes" and B := "Neighbours for the attractant N_1", etc. Some conditions of the experiment, fixed by subsets of Ω^ω, do not change for different time t = 0, 1, 2, … Some other conditions change for different time t = 0, 1, 2, … So, we can see that the property B is verified on the same number of members of Ω for any time t = 0, 1, 2, … Nevertheless, the property A is verified on a different number of members for different time t = 0, 1, 2, … Thus, describing the experiment, we deal not with properties A, B, etc., but with properties A^ω, B^ω, etc. Let us define the cardinality number of $X^\omega \subseteq \Omega^\omega$ as follows: $|X^\omega| := (|X|$ for $t = 0$; $|X|$ for $t = 1$; $|X|$ for $t = 2, ...)$, where $|X|$ means a cardinality number of X. Notice that if $|\Omega| = p - 1$, then $|A^\omega|$, $|B^\omega|$, and $|\Omega^\omega|$ cover p-adic integers.

The simplest way to define p-adic probabilities is as follows:

$$P(A^\omega) = |\, A^\omega \,| \text{ or } P(A^\omega) = |\, A^\omega \,| \,/\, |\, \Omega^\omega \,|$$

Notice that in p-adic metric, $|\, \Omega^\omega \,| = -1$

Agent i's knowledge structure is a function $\mathbf{P}_i$ which assigns to each $a \in \Omega^\omega$ a non-empty subset of Ω^ω, so that each world a belongs to one or more elements of each $\mathbf{P}_i$, i.e. Ω^ω is contained in a union of $\mathbf{P}_i$, but $\mathbf{P}_i$ are not mutually disjoint. The function $\mathbf{P}_i$ is interpreted on p-adic probabilities.

$$K_i A^\omega = \{a : A^\omega \subseteq P_i(\omega)\}$$

The double-slit experiment with *Physarum polycephalum* shows that, first, we cannot extract atomic actions from all the kinds of the plasmodium behaviour, second, probability measures used in describing this experiment are not additive. We can deal just with hybrid actions.

The informal meaning of hybrid actions (e.g. hybrid terms or hybrid formulas) is that any hybrid action is defined just on streams and we cannot say in accordance with which stream the hybrid action will be embodied in the given environment. It can behave like any stream it contains but there is an uncertainty how exactly.

7 CONCLUSIONS & FUTURE WORK

Thus, context-based games on the medium of *Physarum polycephalum* can have many impacts in the development of unconventional computing: from behavioural sciences to quantum computing and many other fields.

So, if we perform the *double-slit experiment* for *Physarum polycephalum*, we detect self-inconsistencies showing that we cannot approximate atomic individual acts of Physarum as well as it is impossible to approximate single photons. From the standpoint of measure theory, it means that we cannot define additive measures for *Physarum* actions. In our opinion, it is a fundamental result for many behavioural sciences. Non-additivity of actions can be expressed in different ways: (i) *natural transition systems, such as Physarum behaviour, cannot be reduced to Kolmogorov-Uspensky machines*, although their actions are intelligent, (ii) *there is an individual-collective duality, when we cannot approximate atomic individual acts* (an individual, such as plasmodium, can behaves like a collective and a collective, such as collective of plasmodia, can behaves like an individual).

ACKNOWLEDGEMENTS

This research is supported by FP7-ICT-2011-8.

REFERENCES

[1] A. Adamatzky, V. Erokhin, M. Grube, Th. Schubert, A. Schumann, A. Physarum Chip Project: Growing Computers From Slime Mould, *Int. J. of Unconventional Computing*, 8(4): 319-323, (2012).

[2] A. Schumann, Payoff Cellular Automata and Reflexive Games, *J. of Cellular Automata*, 9(4): 287-313, (2015).

[3] A. Schumann, L. Akimova, Simulating of Schistosomatidae (Trematoda: Digenea) Behavior by Physarum Spatial Logic, *Annals of Computer Science and Information Systems, Volume 1. Proceedings of the 2013 Federated Conference on Computer Science and Information Systems*. IEEE Xplore, (2013), 225-230.

[4] A. Schumann, K. Pancerz, Towards an Object-Oriented Programming Language for Physarum Polycephalum Computing, [in:] M. Szczuka, L. Czaja, M. Kacprzak (eds.), *Proceedings of the Workshop on Concurrency, Specification and Programming* (CS&P'2013), Warsaw, Poland, September 25-27, (2013), 389-397.