Foundations for the Debugging of Functional Programs

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Programs have Bugs

Even functional programs!

strong type system $\Longrightarrow$ cannot corrupt run-time system

but

- wrong result
- abortion with run-time error
- non-termination

Why Debug Functional Programs Differently?

- No canonical execution model.
  - various reduction semantics (small step, big step)
  - interpreters with environments (explicit substitutions)
  - also denotational semantics
- No sequential execution of statements.
  - evaluation of expressions
  - evaluation of subexpressions is independent

    ```
    f (g 3 4) (h 1 2) (i 5) (j 3 9 3)
    ```

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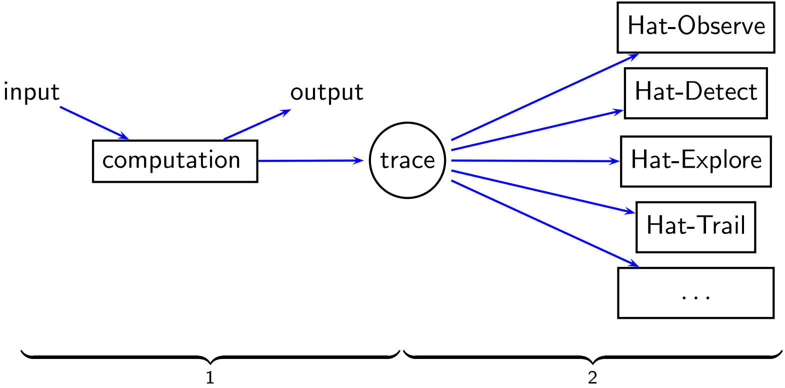
    `f (g 3 4) (h 1 2) (i 5) (j 3 9 3)`

Conclusions

- Abstract from execution details: views for various semantics models.
- Take advantage of simple and compositional semantics.
- Liberate from sequentiality of computation.

The Haskell Tracer Hat (`www.haskell.org/hat`)



Two-Phase Tracing: The trace as data structure.

- Liberates from the time arrow of computation.
- Enables many different views.

But where are formal definitions you can reason with?

Example: Insertion Sort

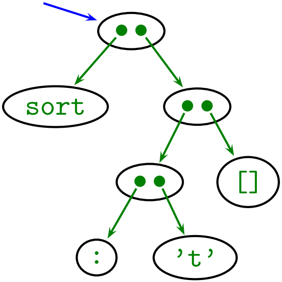
```
main :: String
main = sort "sort"

sort :: Ord a => [a] -> [a]
sort []      = []
sort (x:xs) = insert x (sort xs)

insert :: Ord a => a -> [a] -> [a]
insert x []       = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x:ys
```

There is a bug: `main = "os"` !

The Trace: Simple Graph Rewriting



Start with expression `sort ('t':[])`

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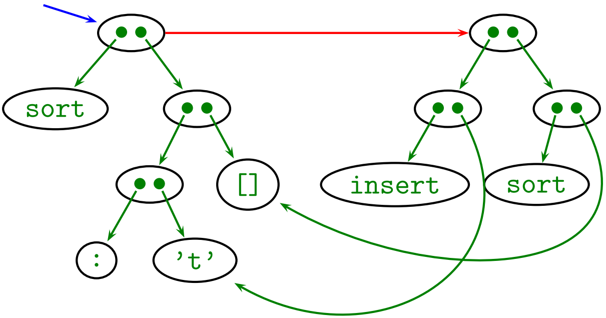


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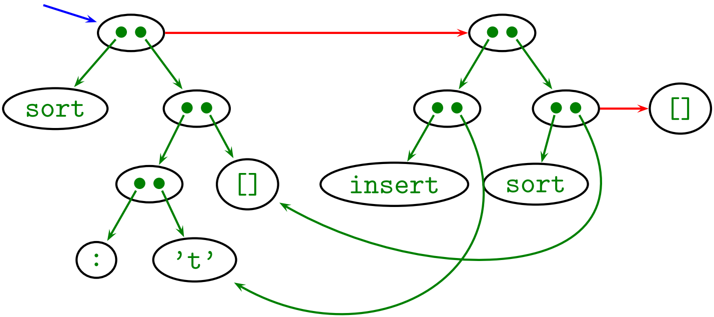


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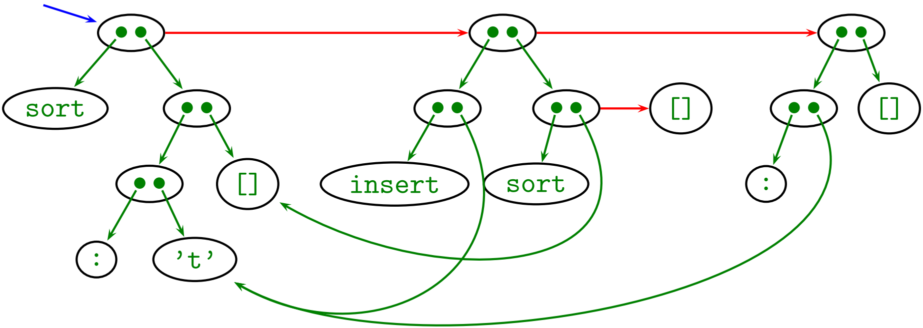


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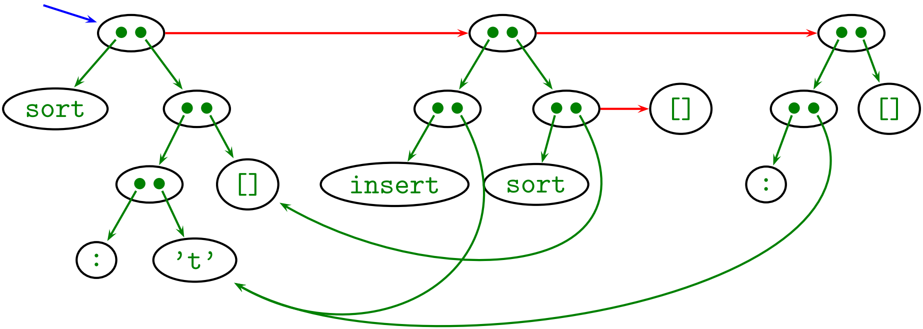
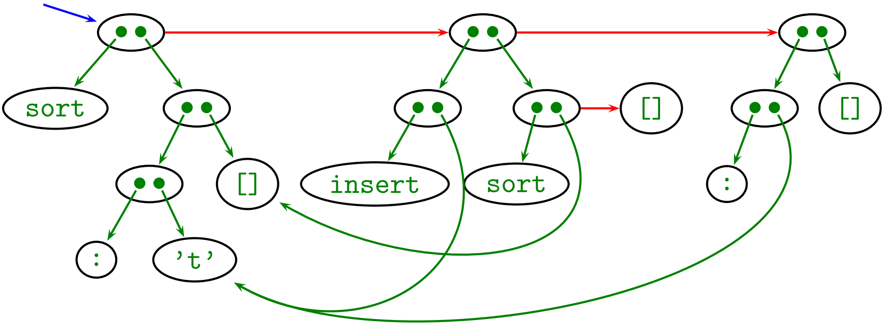


- New nodes for right-hand-side, connected via result pointer.
- Only add to graph, never remove.
- Sharing ensures compact representation.

The Node Naming Scheme



Aim

- not distinguish isomorphic graphs
- avoid inconvenience of isomorphism classes

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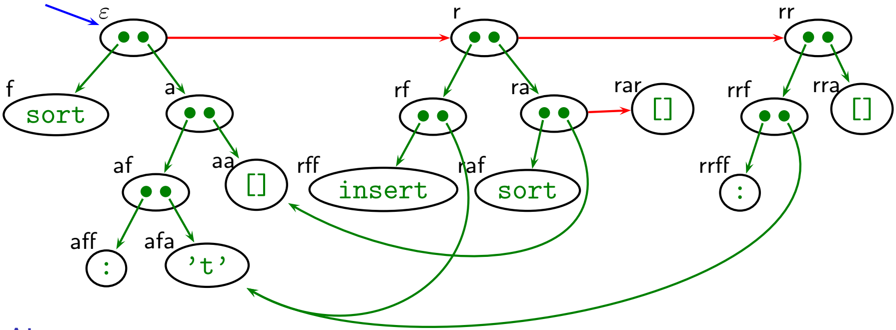

Aim

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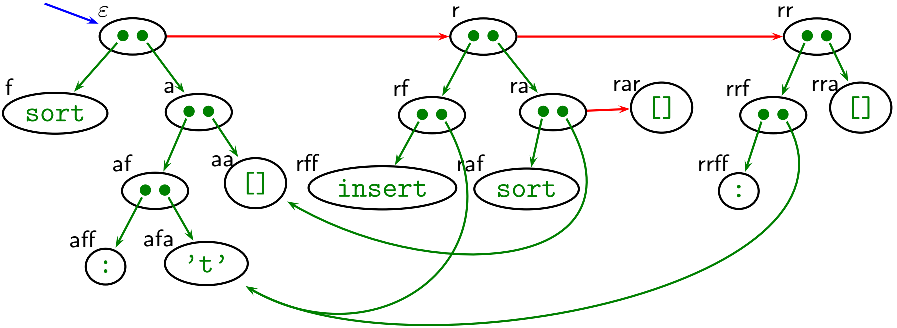
Solution

- standard representation with node describing path from root
- path at creation time (sharing later)
- path independent of evaluation order

The Node Labels



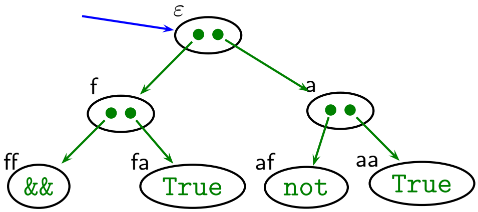
node	n	:=	$\{\mathrm{f}, \mathrm{a}, \mathrm{r}\}^*$	
label term	T	:=	a	atom
		$\mid$	$n\ m$	application of nodes
atom	a	:=	$f \mid C \mid 42 \mid \ldots$	defined variable, data constructor atomic literal, ...

Reduction edge implicitly given through existence of node.

Projections

- Reduction edge implicitly given through existence of node.
- Every redex should be parent of at least one node.
  (otherwise reduction unreachable from computation result)

```
True && x = x
not True = False
```



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$\Rightarrow$ A projection requires an indirection as result.

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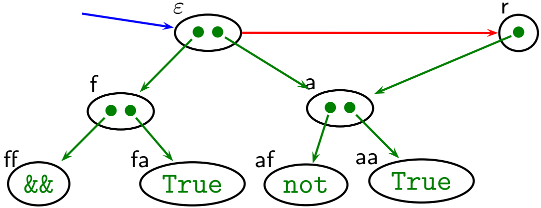

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Projections

- Reduction edge implicitly given through existence of node.
- Every redex should be parent of at least one node.
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⇒ A projection requires an indirection as result.

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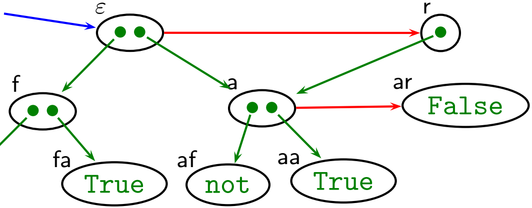

label term	T	$:=$	a	atom
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		$\mid$	n	indirection

atom	a	$:=$	$x \mid C \mid 42 \mid \ldots$	variable, data constructor, …

The Trace: The Augmented Redex Trail (ART)

A trace $\mathcal{G}$ for initial term M and program P is a partial function from nodes to term constructors, $\mathcal{G} : n \mapsto T$, defined by

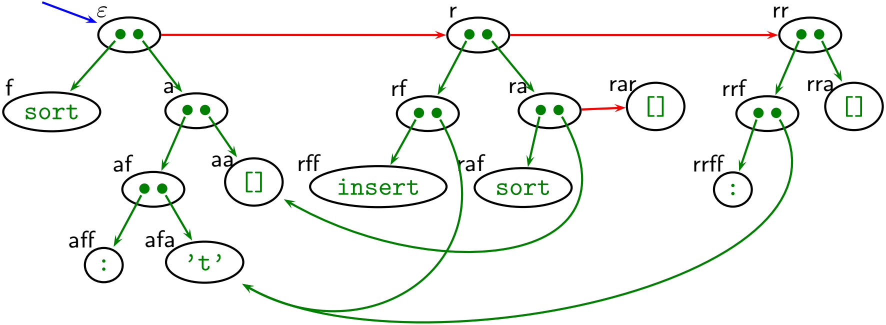
- The unshared graph representation of M, $\text{graph}_{\mathcal{G}}(\varepsilon, M)$, is a trace.
- If $\mathcal{G}$ is a trace and
  - $L = R$ an equation of the program P,
  - σ a substitution replacing argument variables by nodes,
  - $\text{match}_{\mathcal{G}}(n, L\sigma)$,
  - $nr \notin \text{dom}(\mathcal{G})$,

  then $\mathcal{G} \cup \text{graph}_{\mathcal{G}}(nr, R\sigma)$ is a trace.

No evaluation order is fixed.

The Most Evaluated Form of a Node

A node represents many terms, in particular a most evaluated one.



$$\mathrm{mef}_{\mathcal{G}}(\varepsilon) = \texttt{(:) 't' []}$$

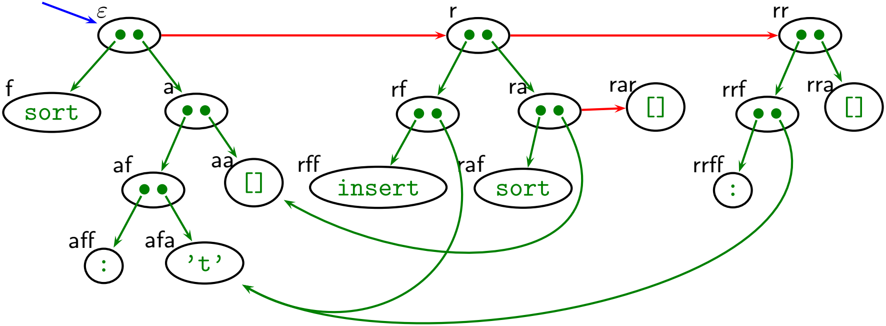
Definition

$$n \succ_{\mathcal{G}} m \quad \Leftrightarrow \quad m = n\mathrm{r} \vee \mathcal{G}(n) = m$$
$$\lceil n \rceil_{\mathcal{G}} = m \quad \Leftrightarrow \quad n \succ^{*}_{\mathcal{G}} m \wedge \nexists o.\, m \succ_{\mathcal{G}} o$$

Definition

$$\mathrm{mef}_{\mathcal{G}}(n) = \mathrm{mefT}_{\mathcal{G}}(\mathcal{G}(\lceil n \rceil_{\mathcal{G}}))$$
$$\mathrm{mefT}_{\mathcal{G}}(a) = a$$
$$\mathrm{mefT}_{\mathcal{G}}(n) = \mathrm{mef}_{\mathcal{G}}(n)$$
$$\mathrm{mefT}_{\mathcal{G}}(n\, m) = \mathrm{mef}_{\mathcal{G}}(n)\ \mathrm{mef}_{\mathcal{G}}(m)$$

Redexes and Big-Step Reductions



$\text{redex}_{\mathcal{G}}(\text{r}) = \texttt{insert 't' []}$

$\text{bigstep}_{\mathcal{G}}(\text{r}) = \texttt{insert 't' [] = (:) 't' []}$

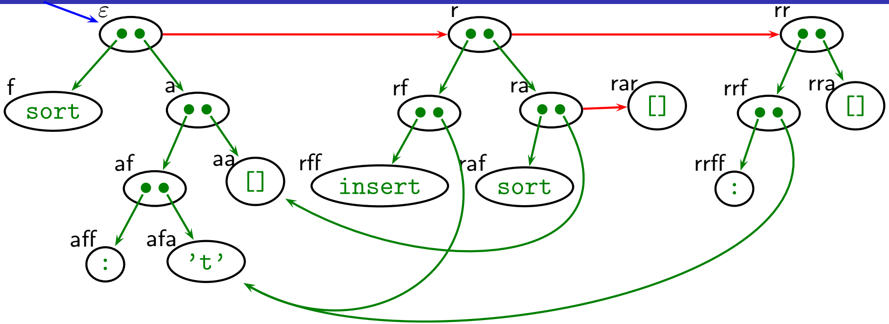
Definition

For any redex node n, i.e., $nr \in \text{dom}(G)$

$$\text{redex}_{\mathcal{G}}(n) = \begin{cases} \text{mef}_{\mathcal{G}}(m)\ \text{mef}_{\mathcal{G}}(o) & \text{, if } \mathcal{G}(n) = m\ o \\ a & \text{, if } \mathcal{G}(n) = a \end{cases}$$

$$\text{bigstep}_{\mathcal{G}}(n) = \text{redex}_{\mathcal{G}}(n) = \text{mef}_{\mathcal{G}}(n)$$

From Trace to Big-Step Computation Tree



ε `sort ('t':[]) = 't':[]`

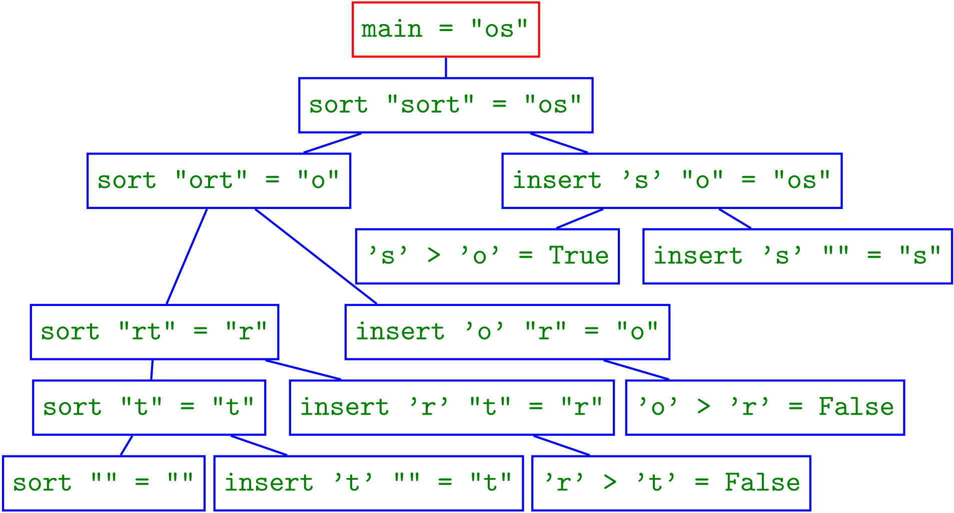
ra `sort [] = []`

r `insert 't' [] = 't':[]`

- Every redex node n yields a tree node n labelled $\mathrm{bigstep}_{\mathcal{G}}(n)$.
- Tree node n is child of tree node $\mathrm{parent}(n)$.

$$\begin{aligned}\mathrm{parent}(nr) &= n\\ \mathrm{parent}(nf) &= \mathrm{parent}(n)\\ \mathrm{parent}(na) &= \mathrm{parent}(n)\\ \mathrm{parent}(\varepsilon) &= \mathrm{undefined}\end{aligned}$$

Algorithmic Debugging with the Computation Tree



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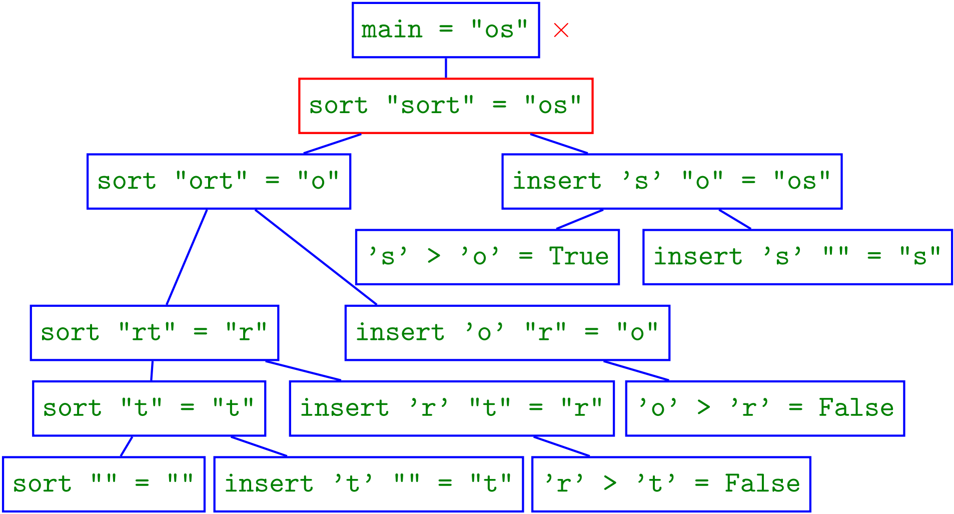

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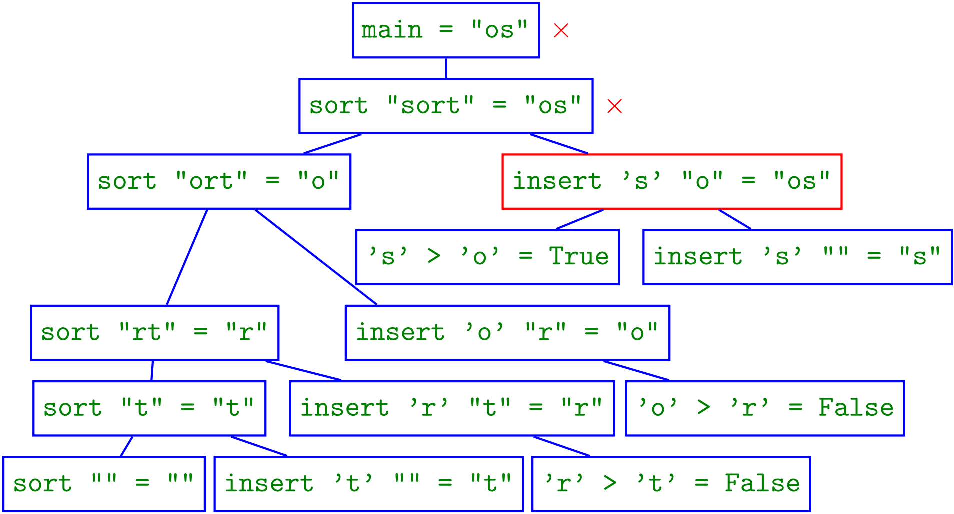

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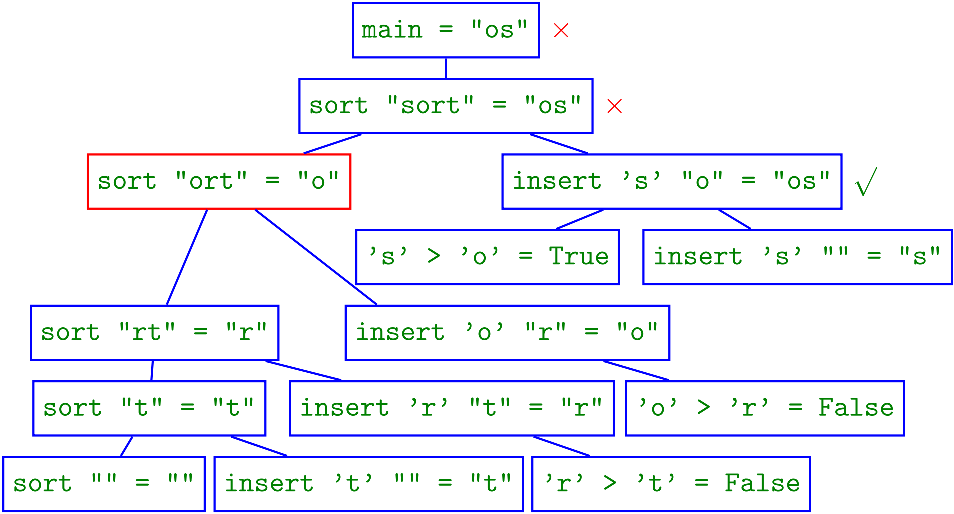

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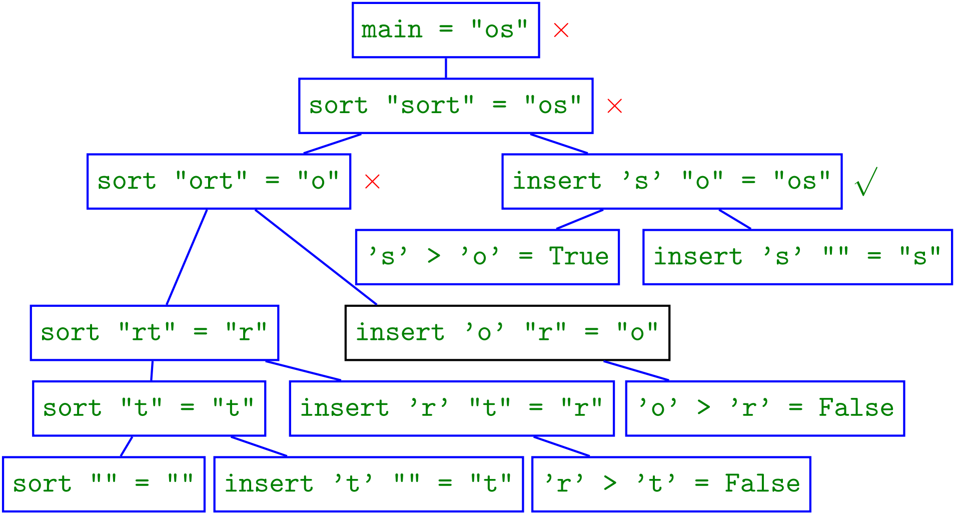

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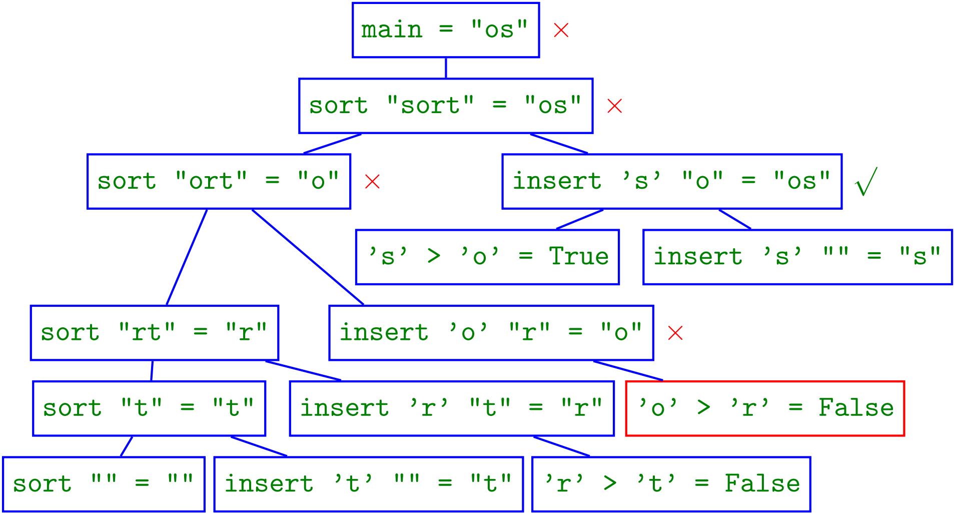

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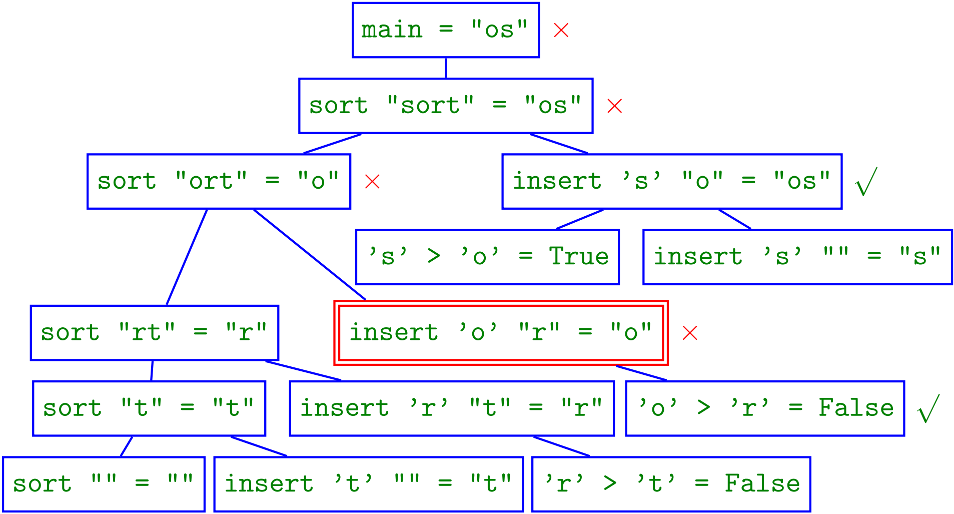


Fault located!

```
main :: String
main = sort "sort"

sort :: Ord a => [a] -> [a]
sort []     = []
sort (x:xs) = insert x (sort xs)

insert :: Ord a => a -> [a] -> [a]
insert x []     = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x:ys
```

Faulty computation: `insert 'o' "r" = "o"`

Correctness of Algorithmic Debugging: The Property

If node n incorrect and all its children correct, then node n faulty, i.e., its equation is faulty.

ε `sort ('t':[]) = 't':[]`

ra `sort [] = []`

r `insert 't' [] = 't':[]`

Definition

Tree node n incorrect $\Leftrightarrow$ $\mathrm{redex}_{\mathcal{G}}(n) \not\cong_{\mathrm{I}} \mathrm{mef}_{\mathcal{G}}(n)$.

Tree node n faulty $\Leftrightarrow$ $\mathrm{redex}_{\mathcal{G}}(n) \not\cong_{\mathrm{I}} \mathrm{reduct}_{\mathcal{G}}(n)$.

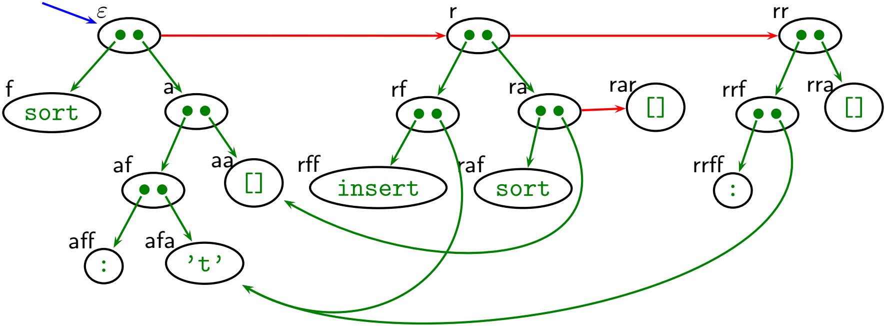
If tree node n faulty, then for its program equation $L = R$ exists substitution σ such that $L\sigma \not\cong_{\mathrm{I}} R\sigma$.

Soundness of Algorithmic Debugging: Main Theorem

Theorem

Let n be a redex node. If for all redex nodes m with $parent(m) = n$ we have $\mathrm{redex}_{\mathcal{G}}(m) \cong_I \mathrm{mef}_{\mathcal{G}}(m)$, then $reduct_{\mathcal{G}}(n) \cong_I \mathrm{mef}_{\mathcal{G}}(n)$.

With $\mathrm{redex}_{\mathcal{G}}(n) \ncong_{\mathrm{I}} \mathrm{mef}_{\mathcal{G}}(n)$ follows $\mathrm{redex}_{\mathcal{G}}(n) \ncong_{\mathrm{I}} \mathrm{reduct}_{\mathcal{G}}(n)$.



Higher-Order Insertion Sort

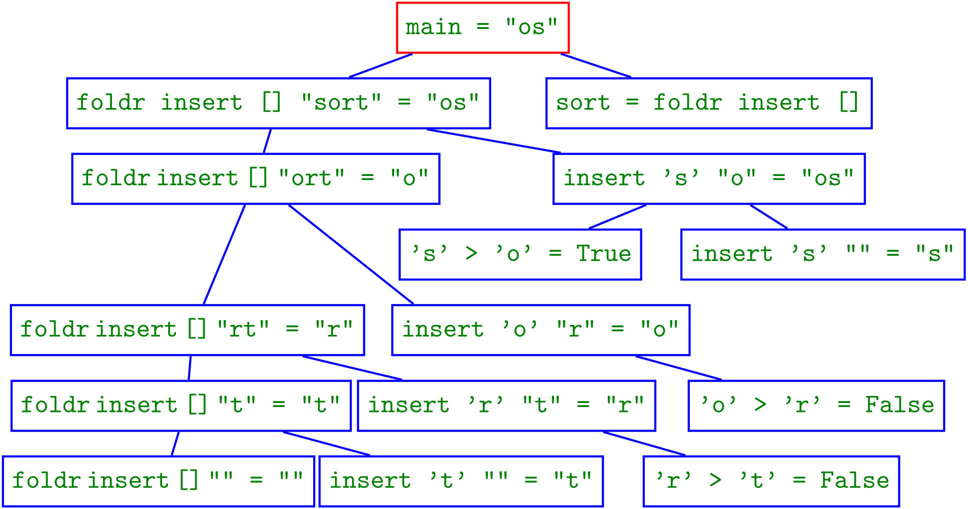
```
main :: String
main = sort "sort"

sort :: Ord a => [a] -> [a]
sort = foldr insert []

foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f a []     = a
foldr f a (x:xs) = f x (foldr f a xs)

insert :: Ord a => a -> [a] -> [a]
insert x []     = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x:ys
```

Higher-Order Algorithmic Debugging



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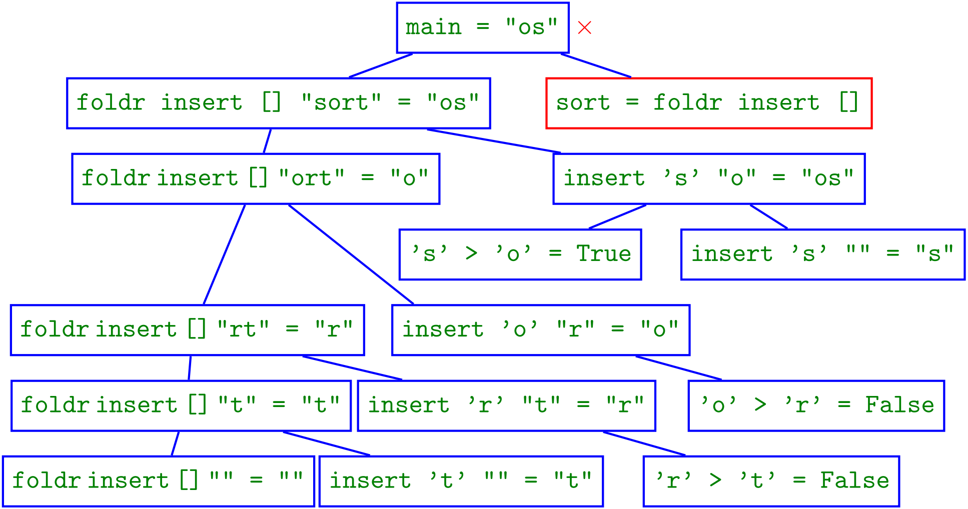

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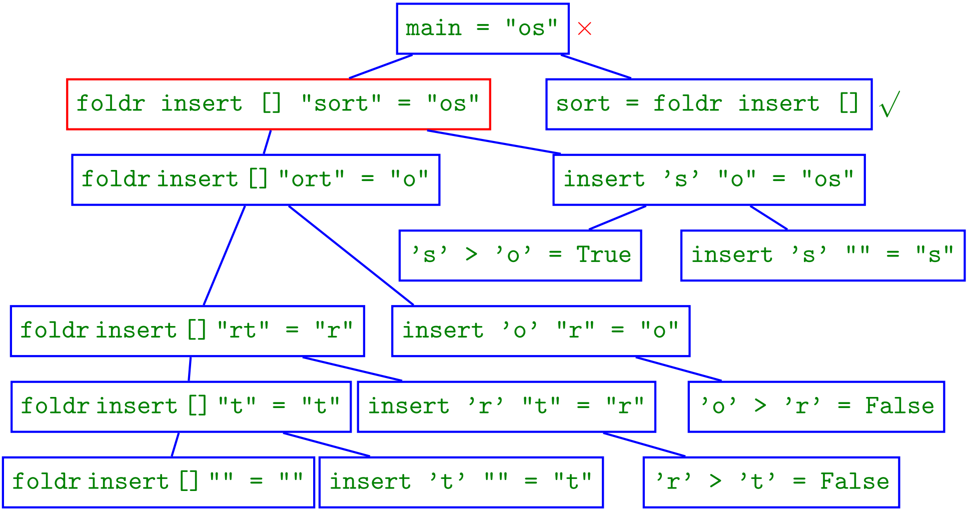

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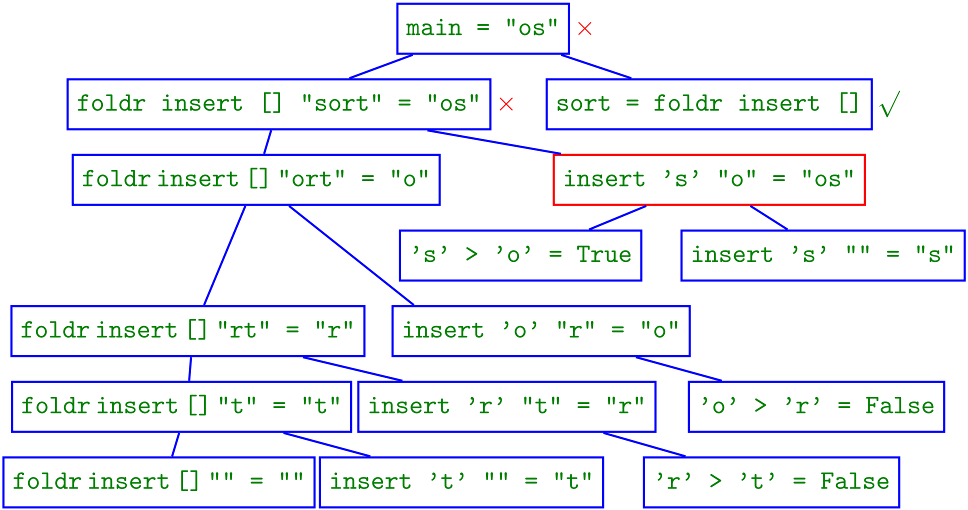

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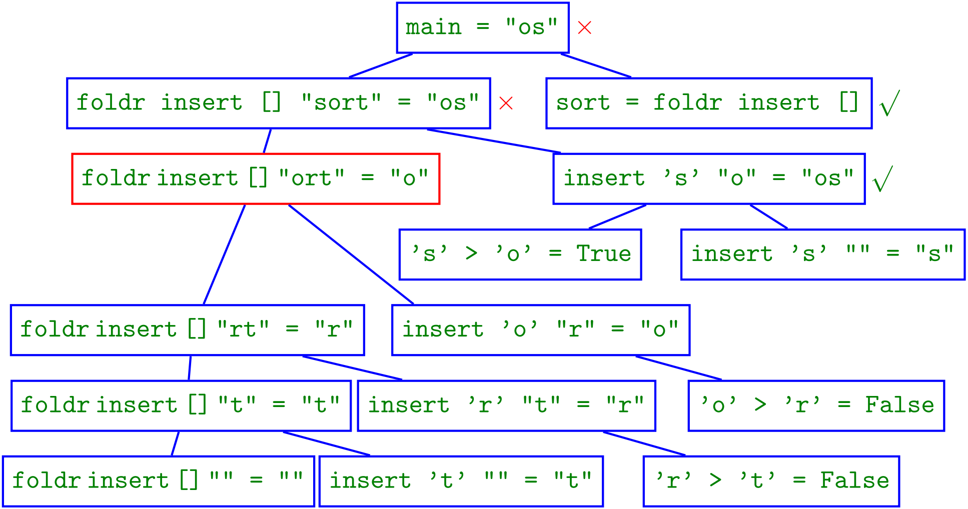

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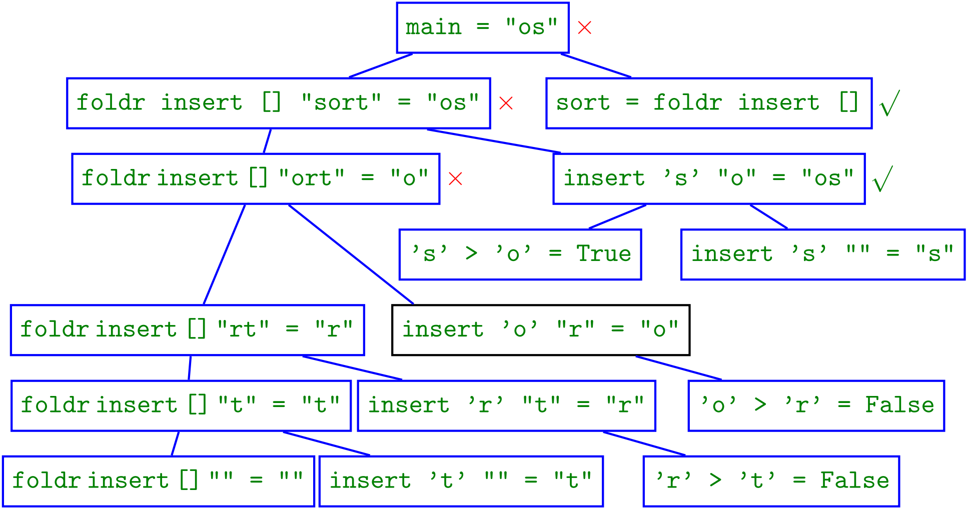

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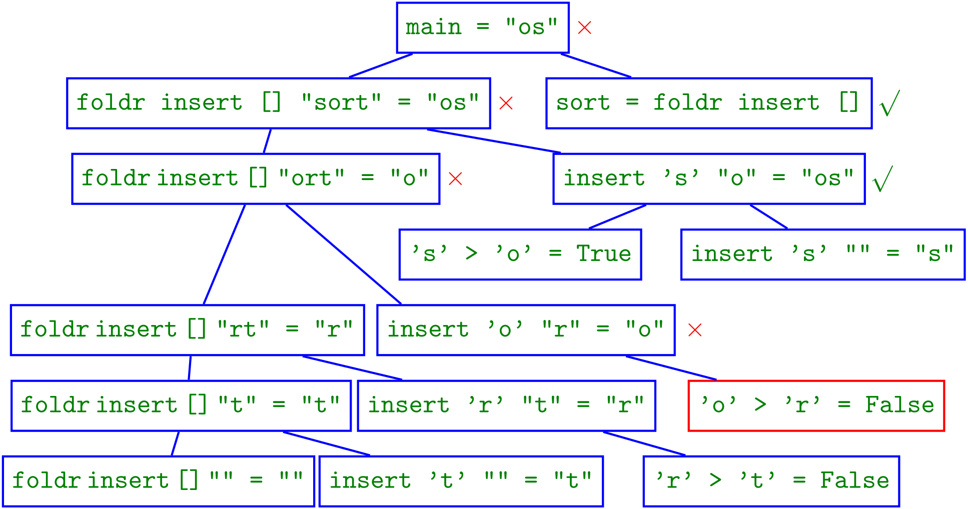

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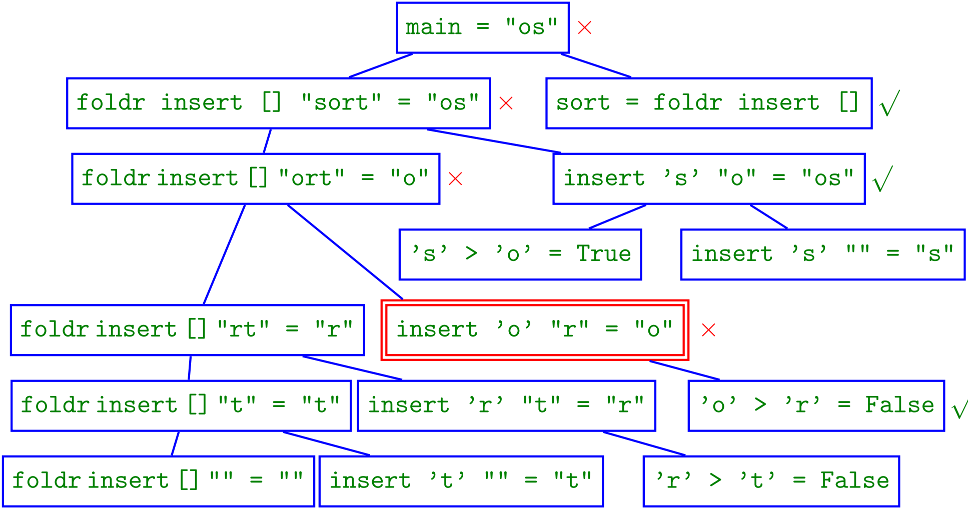
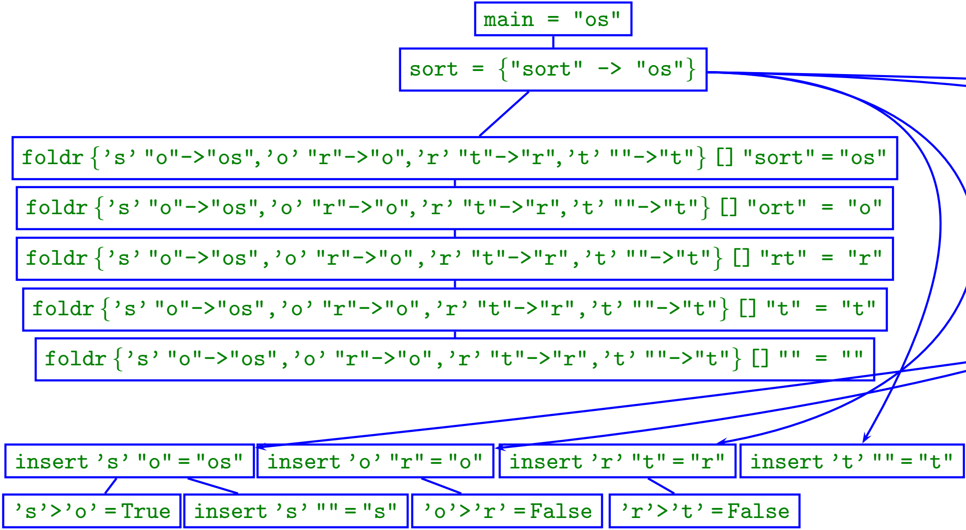


Higher-Order Algorithmic Debugging II



Modify a Few Definitions I

Definition (Most evaluated form for finite maps)

$$\mathrm{mef}^{\mathrm{M}}_{\mathcal{G}}(n) = \begin{cases} \mathrm{fMap}_{\mathcal{G}}(n) & \text{, if } M = f\ N_1 \ldots N_k \wedge 0 \le k < \mathrm{arity}(f) \\ \{\} & \text{, if } M = f\ N_1 \ldots N_k \wedge k \ge \mathrm{arity}(f) \\ M & \text{, otherwise} \end{cases}$$
$$\text{where } M = \mathrm{mea}_{\mathcal{G}}(n)$$

$$\mathrm{mea}_{\mathcal{G}}(n) = \mathrm{meaT}_{\mathcal{G}}(\mathcal{G}(\lceil n \rceil_{\mathcal{G}}))$$
$$\mathrm{meaT}_{\mathcal{G}}(a) = a$$
$$\mathrm{meaT}_{\mathcal{G}}(m\, n) = \mathrm{mea}_{\mathcal{G}}(m)\ \mathrm{mef}^{\mathrm{M}}_{\mathcal{G}}(n)$$

$$\mathrm{fMap}_{\mathcal{G}}(n) = \{\mathrm{mef}^{\mathrm{M}}_{\mathcal{G}}(o) \mapsto \mathrm{mef}^{\mathrm{M}}_{\mathcal{G}}(m) \mid \mathcal{G}(m) = n'\ o \wedge n' \succ^{*}_{\mathcal{G}} n \wedge \mathrm{mef}^{\mathrm{M}}_{\mathcal{G}}(m) \neq \{\}\}$$

Modify a Few Definitions II

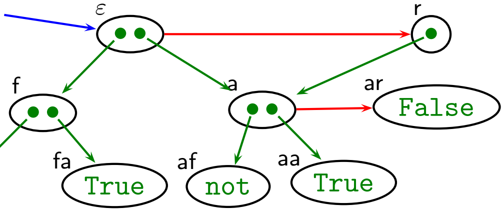
Definition (Parent for finite maps)

$$\mathsf{parentFDT}_{\mathcal{G}} = \mathsf{parent} \cdot \mathsf{fun}_{\mathcal{G}}$$

$$\mathsf{fun}_{\mathcal{G}}(n) = \begin{cases} n & \text{, if } \mathcal{G}(n) = a \\ \mathsf{fun}_{\mathcal{G}}(\lceil m \rceil_{\mathcal{G}}) & \text{, if } \mathcal{G}(n) = m\, o \end{cases}$$

Conclusions

- Simple model amenable to proof.
- Contains a wealth of information about computation.
- Models real-world trace of Haskell tracer Hat.
- Proves soundness of algorithmic debugging.



http://www.haskell.org/hat
http://www.cs.kent.ac.uk/people/staff/oc/traceTheory.html