

Turing's



Universal

Digital

Computer

Why Universal

Predecessors

Self reference

Turing proved self-reference in order to solve the halting problem. But this generally means the machine is not a model of a real-world machine. It is a model of a machine that can simulate any machine. The only benefit of digital computers, that they can simulate any machine, is to be used to simulate any machine.

Cheapness

The universal machine has the advantage that one design can be scaled to any problem. This is a huge economy of scale in manufacturing.

Digital

Reality is digital, Turing computability rules.

Why Computer

Universal Computer Equivalent to Lambda Calculus?

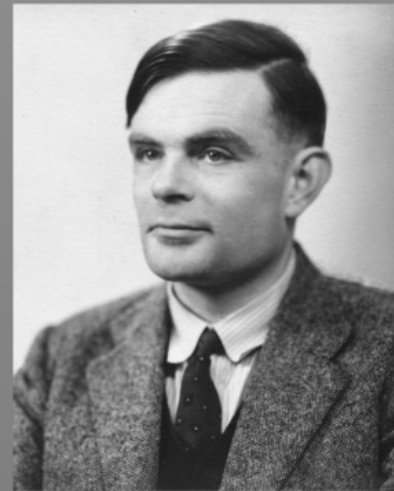
Pilot Ace Console

Turing's idea was thus something much more fundamental and lasting than that of Church

Turing's

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Digital



Computer

rsal

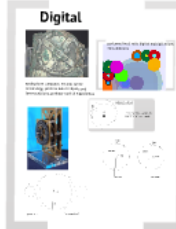
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Why Universal

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Self reference

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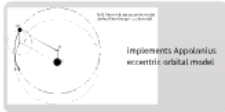
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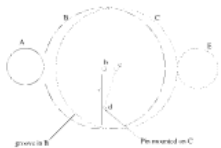
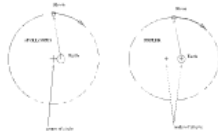
Antikythera computer, ancient Greek technology, predicts lunar eclipses and lunar positions, perhaps work of Apollonius



performs fixed ratio digital multiplications and additions



Implements Apollonius' eccentric orbital model



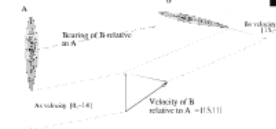
Analogue



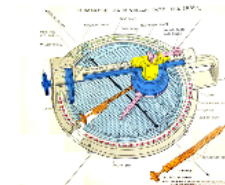
100 years ago navies had to solve real-time vector arithmetic problems and regression problems that were way beyond what could be done manually.



Ranges were about 10 miles
relative velocities up to 60 mph
time of flight of the order 20secs



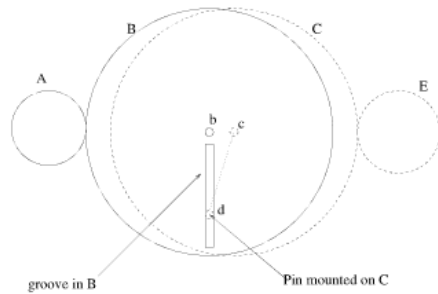
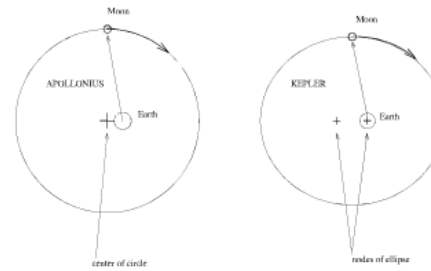
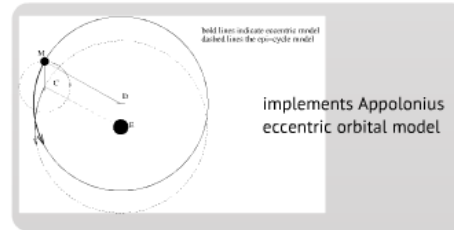
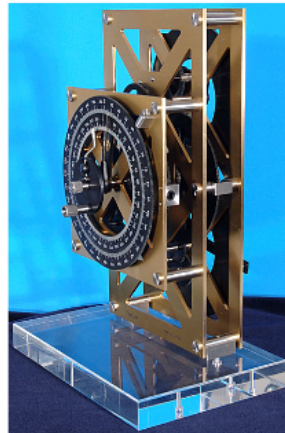
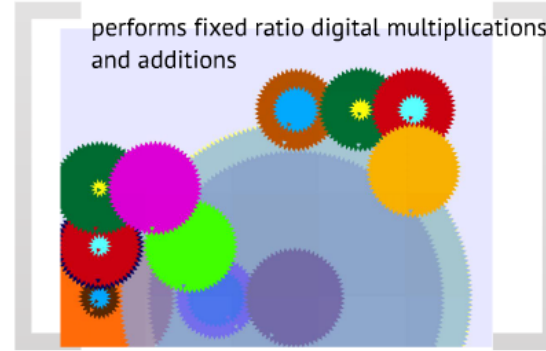
Rear Admiral John Dumasq
commander of Royal Australian
Navy described by his colleagues in
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genius, and unsurprisingly pioneer of
mechanical computing



Digital



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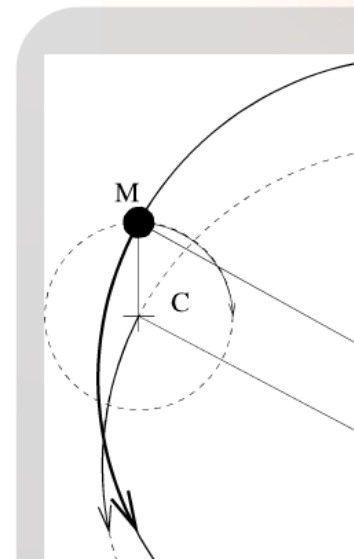


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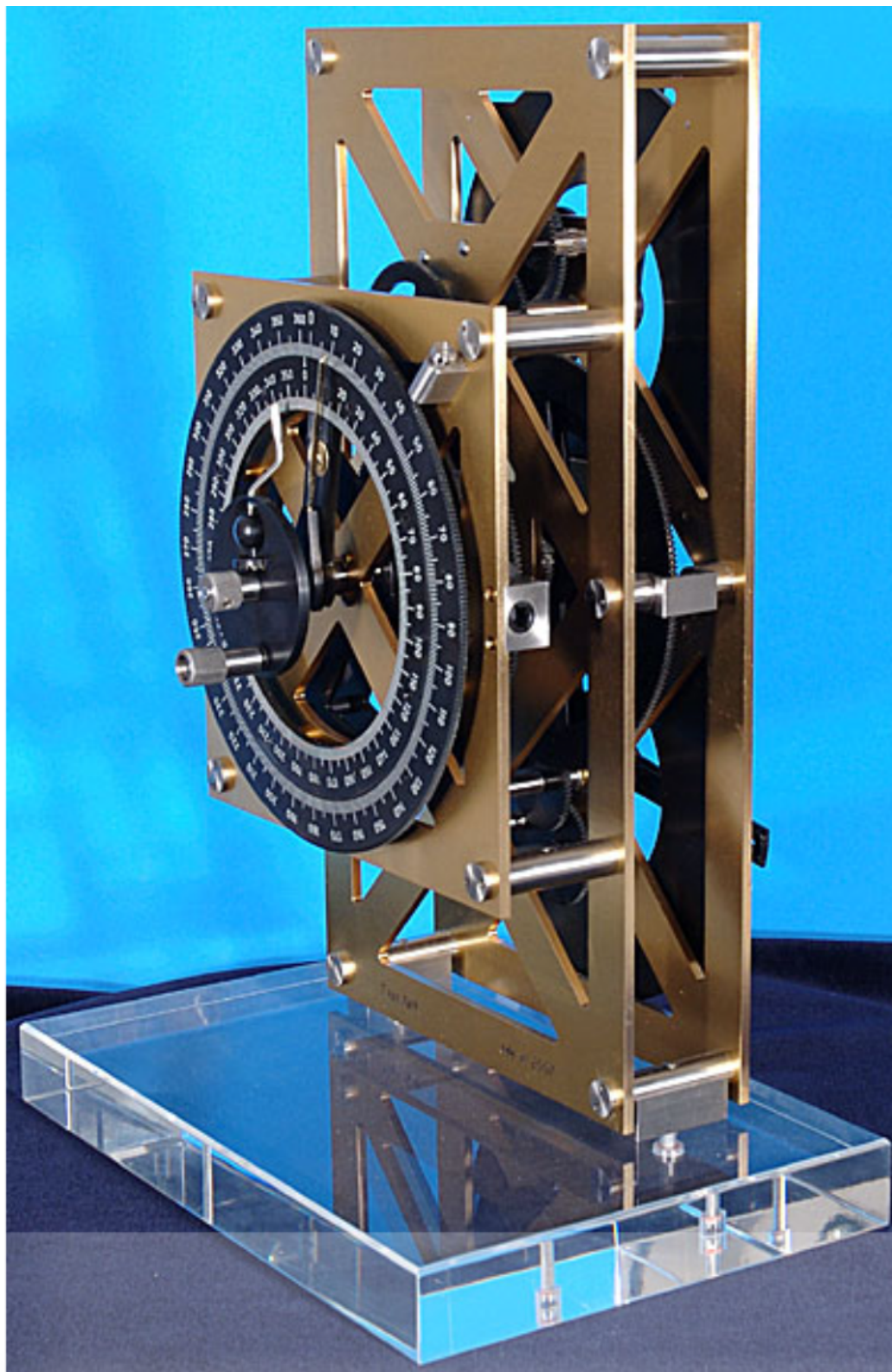


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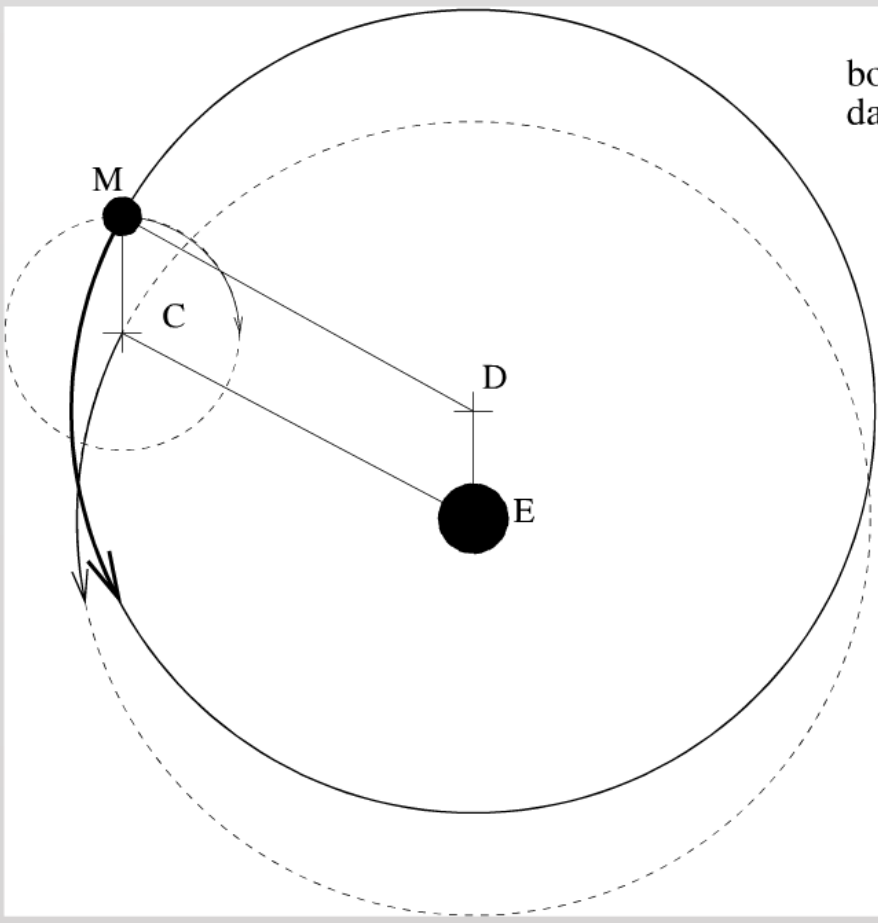


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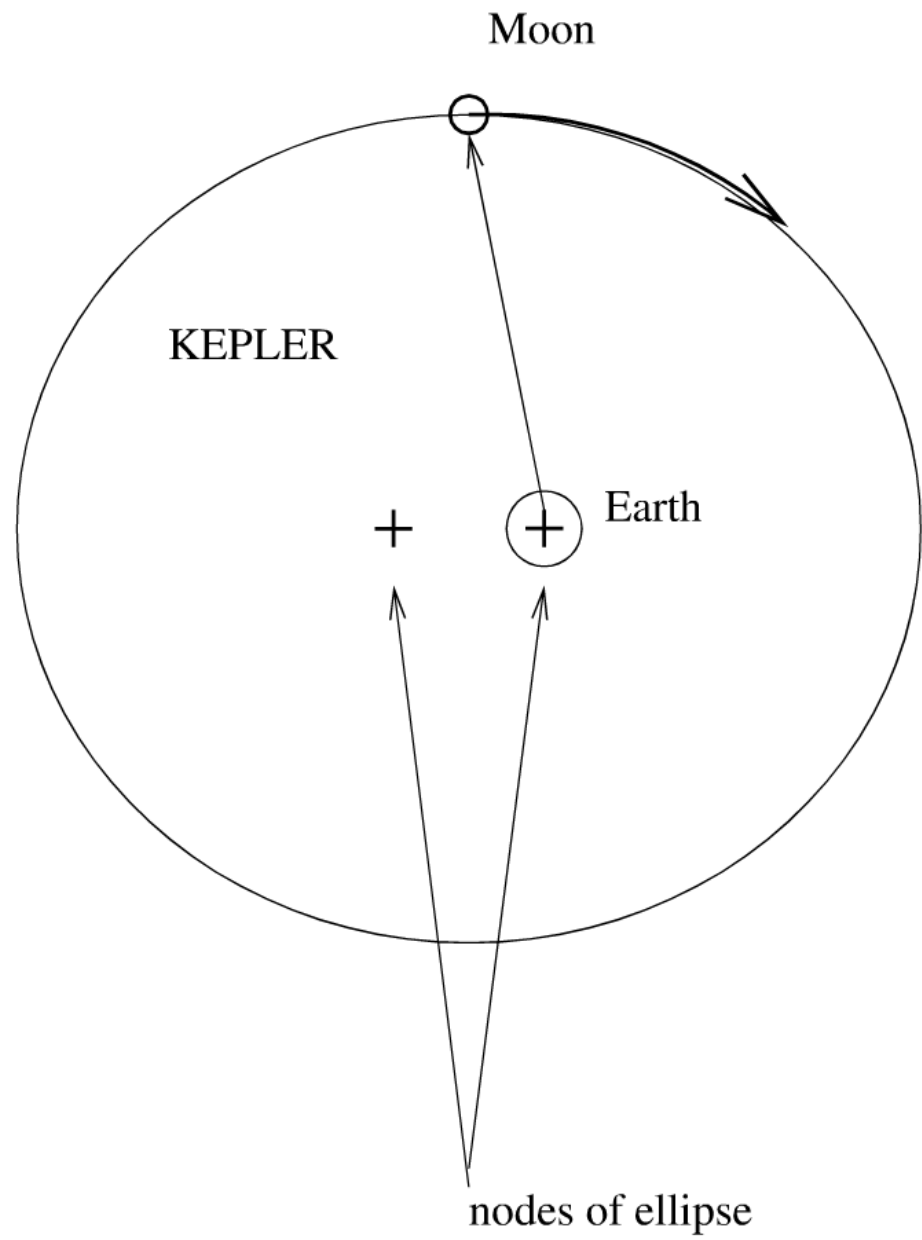
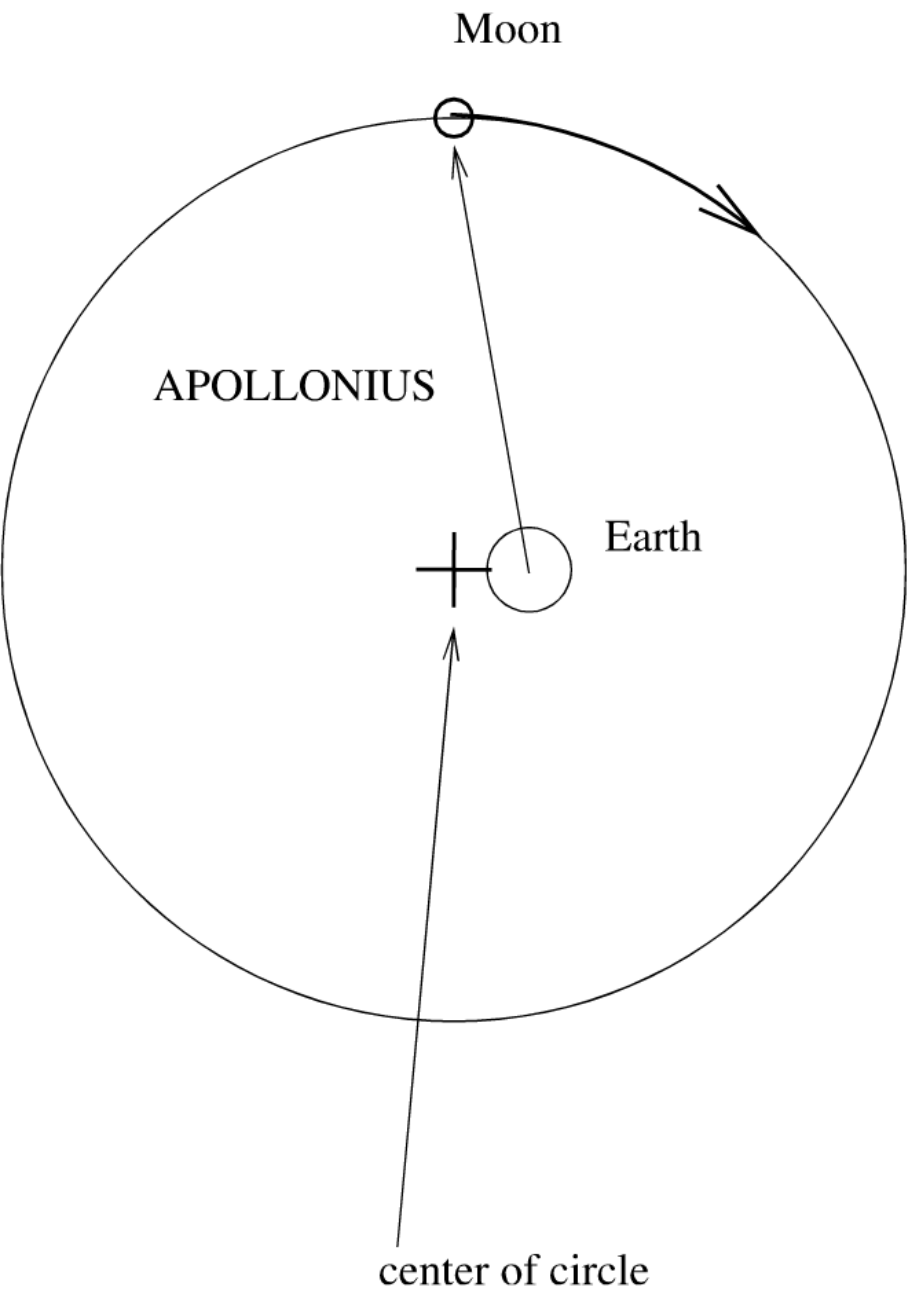


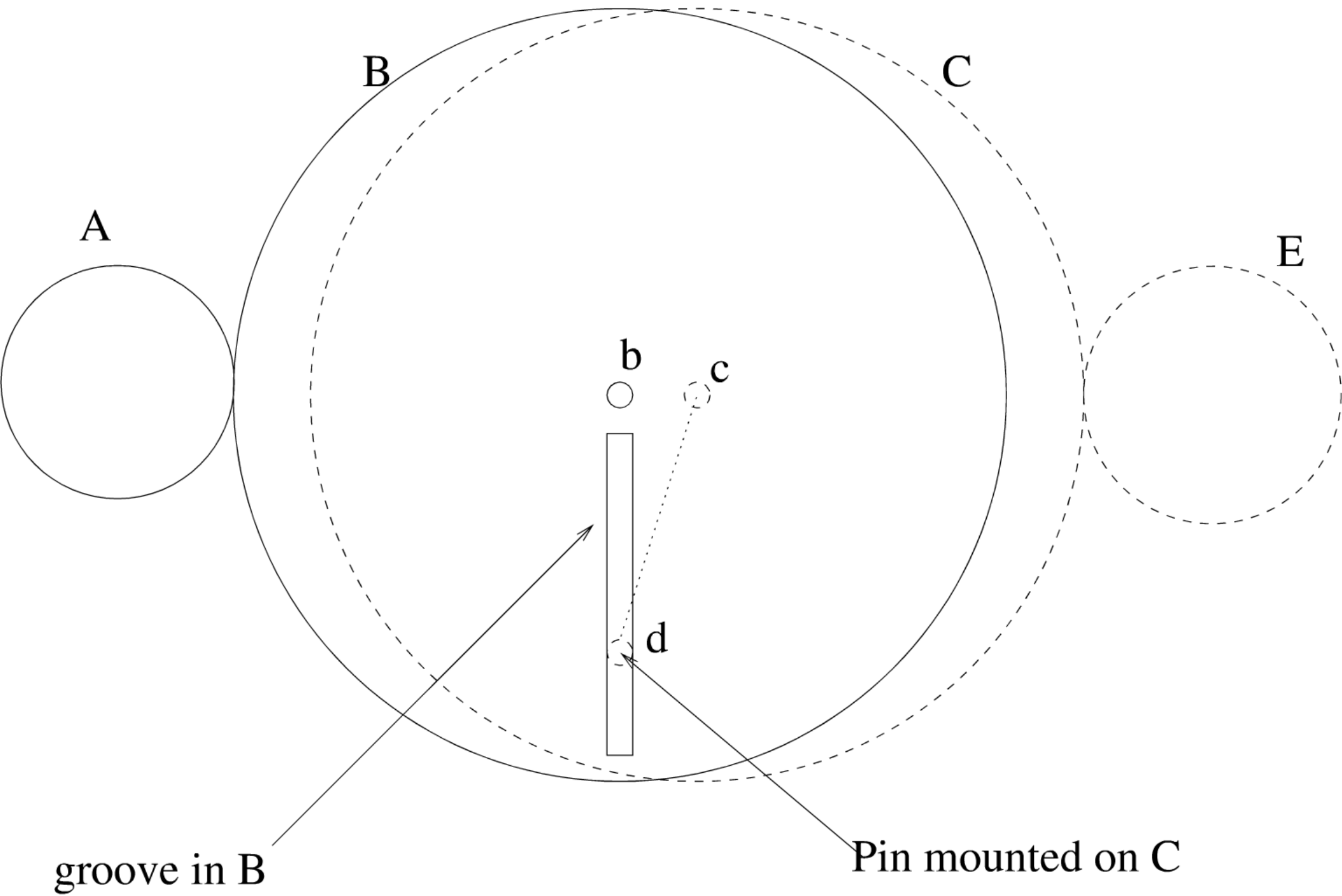
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bold lines indicate eccentric model
dashed lines the epi-cycle model

implements Appolonius
eccentric orbital model





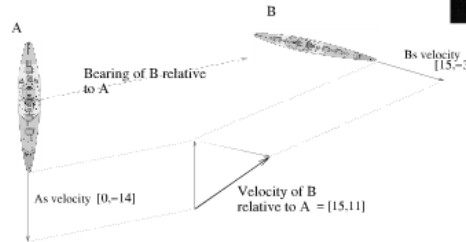
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Analogue

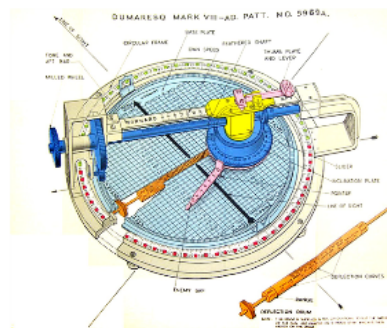


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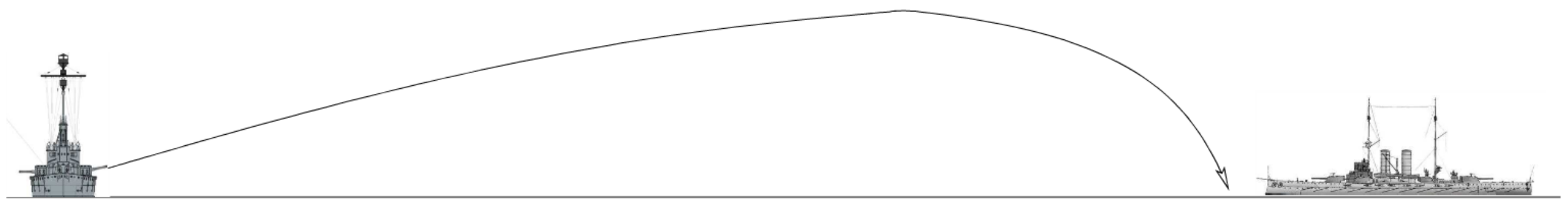
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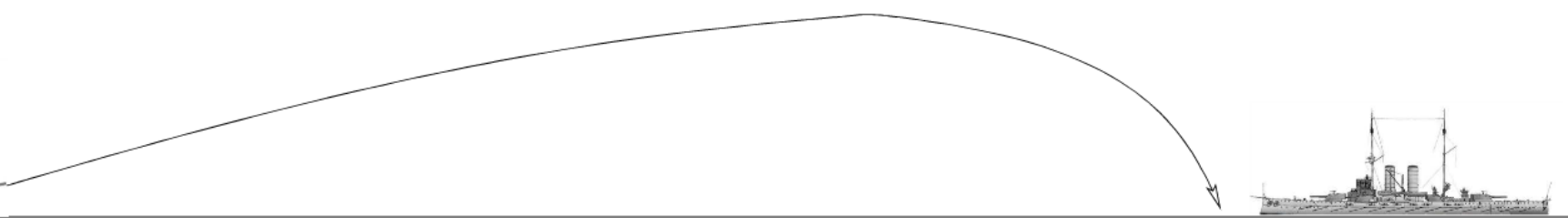


Pythagore

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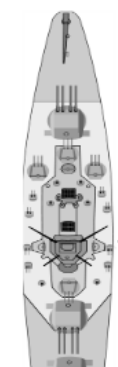






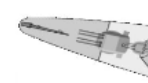
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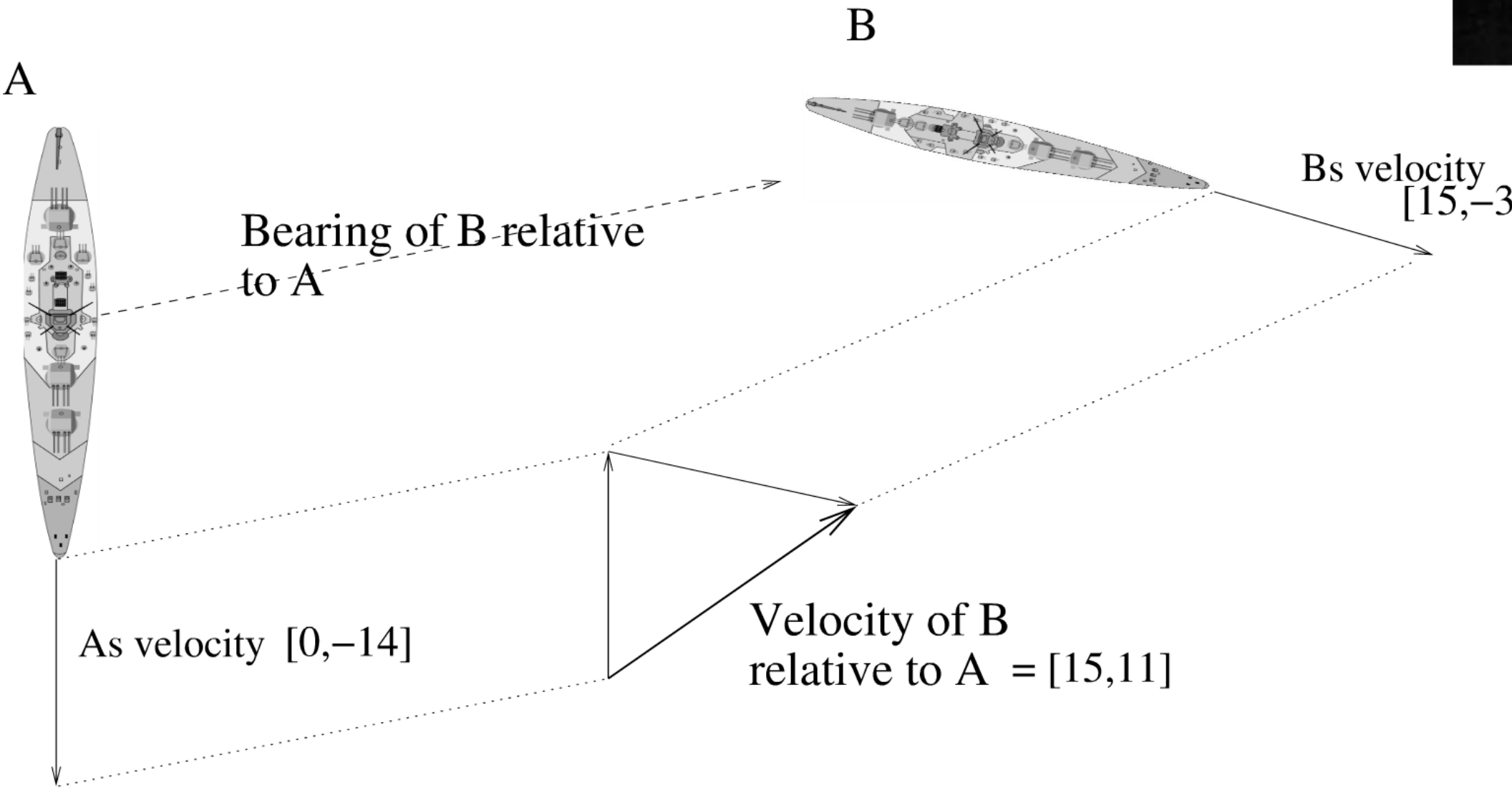
Bearing of B relative
to A

B





f the order 20secs

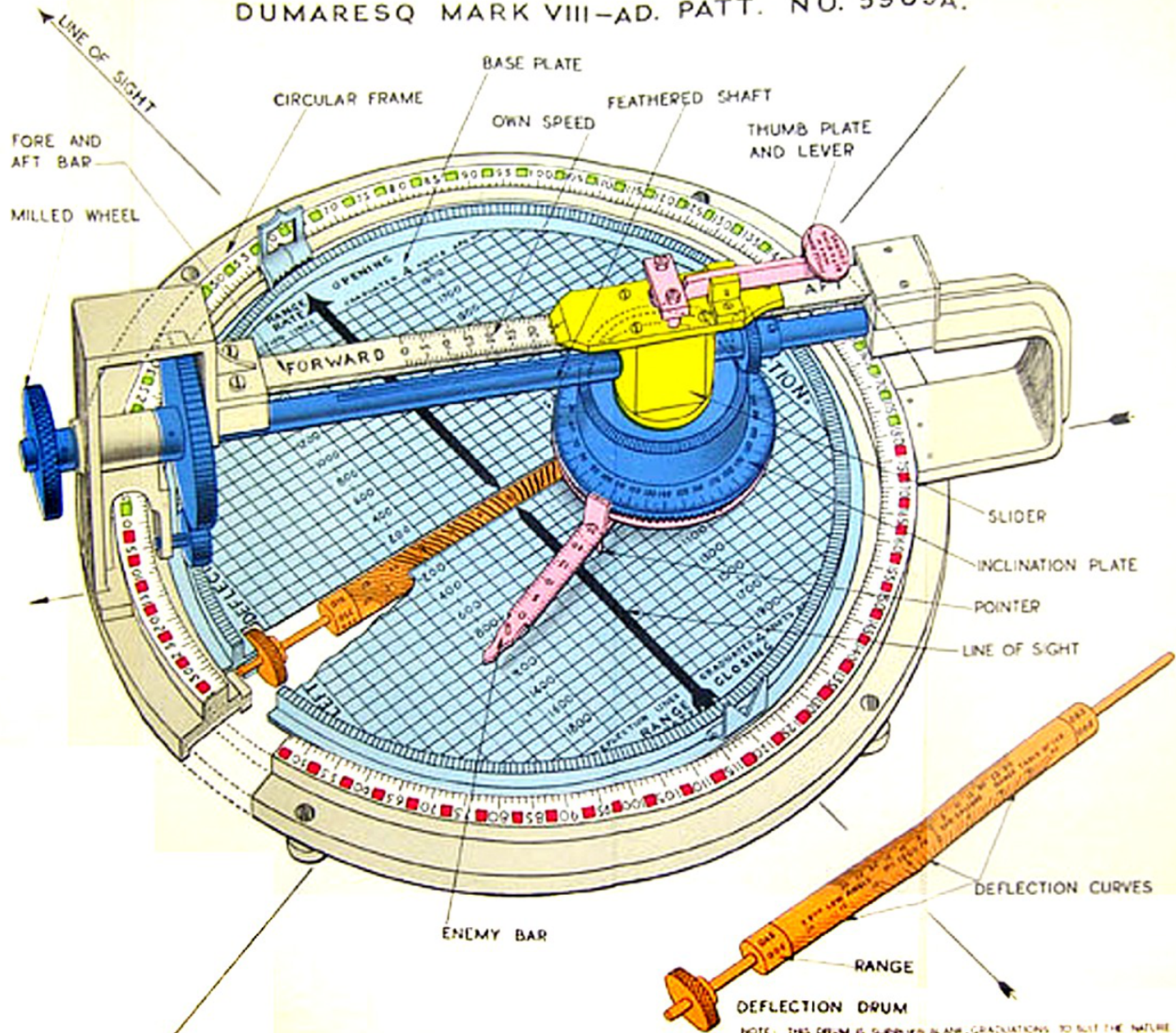


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DUMARESQ MARK VIII-AD. PATT. NO. 5969A.



NOTE: THIS DRUM IS SUPPLIED BLANK, GRADUATIONS TO SUIT THE NATURE OF THE GUN, ARE PRINTED ON A PAPER STRIP WHICH IS THEN PASTED ON THE DRUM.

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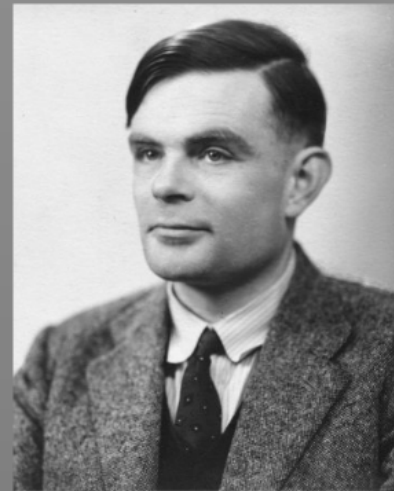
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That the machine is digital however has a more subtle significance. It means firstly that numbers can be represented by strings of digits that can be as long as one wishes. One can therefore work to any desired degree of accuracy. This accuracy is not obtained by more careful machining of parts, control of temperature variations and such means, but by a slight increase in the amount of equipment in the machine. To double the number of significant figures, would involve increasing the amount of the equipment by a factor definitely less than two, and would also have some effect in increasing the time taken over each job. This is in sharp contrast with analogue machines, and continuous variable machines such as the differential analyser, where each additional decimal digit required necessitates a complete redesign of the machine, and an increase in the cost by as much as a factor of 10. (Turing, Lecture on the Automatic Computing Engine)

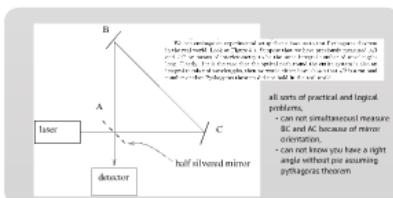
Turings argument there is very pragmatic. There has been a recent temptation to think that we can outperform digital computation by reverting to analogue computing.

This is based on a philosophical misconception that in reality everything is continuous, but we know this is false, everything is digital or quantised.

The notion of the continuum arose in classical Greek geometry from the proof of the irrationality of the length of the hypotenuse of a right triangle with unit sides.

If one assumes, as the Greeks did, that classical geometry was a true theory of the real world, this implied that space must be continuously subdivisible.

But do we know if Pythagoras theorem actually works in the real world? Could we experimentally test it?



More generally there is a fundamental limit to spatial accuracy provided by the Planck length of around 10^{-35} meters which limits the fundamental accuracy of any analogue computing device.

Most proposals for trans Turing computing are based on the illusion that real numbers are 'real' in the ontological sense.

They are based on continuum models of the world like Maxwells equations or Newtonian mechanics.

In the post-Turing era we have to see theories like Maxwells equations or Newtonian mechanics as software packages for making predictions about reality. When combined with a computer to do the maths they allow us to build models that mimic some part of reality.

But just like the Antikythera model, there are limits to the accuracy of our software packages. But our software packages are not reality itself.

Suppose that using some continuum model of mechanics we show that a system with computable boundary conditions has some points where some parameters are uncomputable, does this tell us that the real world can do things which are uncomputable?

No. It tells you that the software package, or physical theory you are using has a bug in it.

It was just such a 'bug' in Maxwells equations, the ultraviolet catastrophe, that let Einstein to invent the quantum theory.

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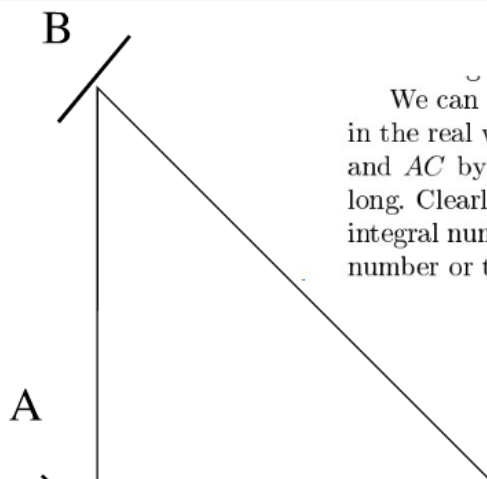
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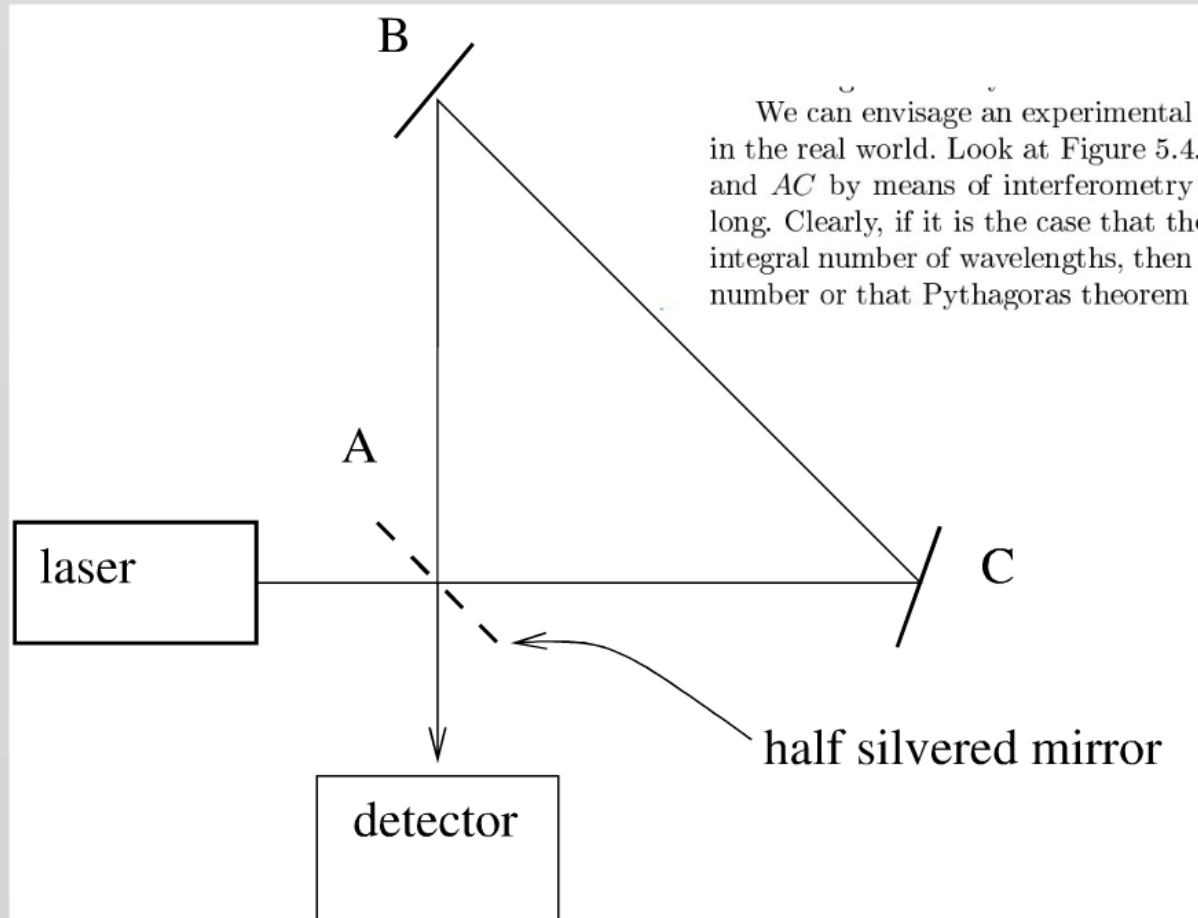
But do we know if Pythagoras theorem actually works in the real world? Could we experimentally test it?



We can envisage an experimental setup that allows us to test Pythagoras theorem in the real world. Look at Figure 5.4. Suppose that we have previously measured AB and AC by means of interferometry to be the same integral number of wavelengths long. Clearly, if it is the case that the optical path round the entire system is also an integral number of wavelengths, then we would either have shown that $\sqrt{2}$ is a rational number or that Pythagoras theorem did not hold in the real world.

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all sorts of practical and logical problems,

- can not simultaneously measure BC and AC because of mirror orientation,
- can not know you have a right angle without pre assuming pythagoras theorem

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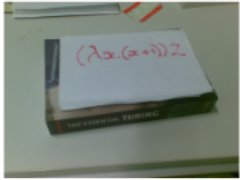


Why Computer

Universal Computer Equivalent to Lambda Calculus?

No!

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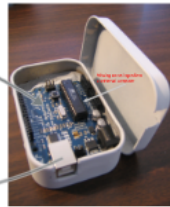


But put into the Lambda Can it worked

Type a term, Help for info or Quit to exit.

```
> (lambda (x) (x+1)) 2
```

```
3  
(7 reductions, 0.00s CPU)  
lci>
```



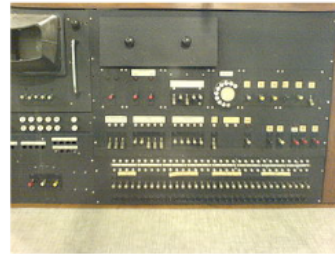
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- A Lambda interpreter on a Universal Computer
- Or a Mathematician, a blackboard and a definition of the calculus that the mathematician understands.

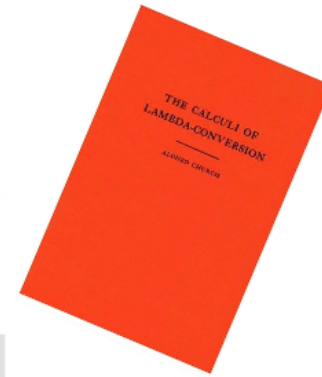
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By introducing a Machine he introduces MECHANICS and indirectly Physics as a support for mathematics.

Pilot Ace Console



This was something real, not a thought machine.



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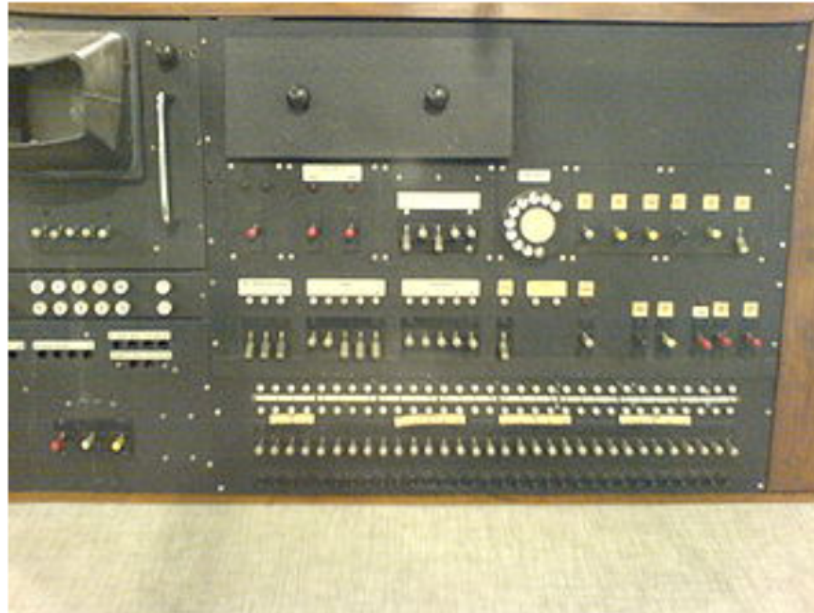
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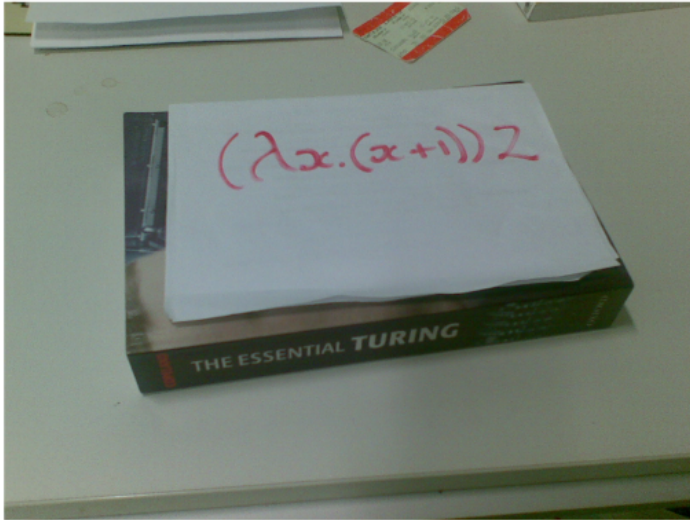
THE CALCULI OF
LAMBDA-CONVERSION

ALONZO CHURCH

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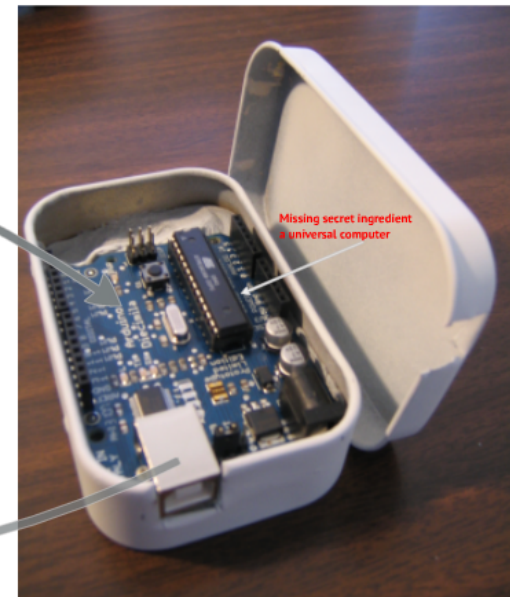
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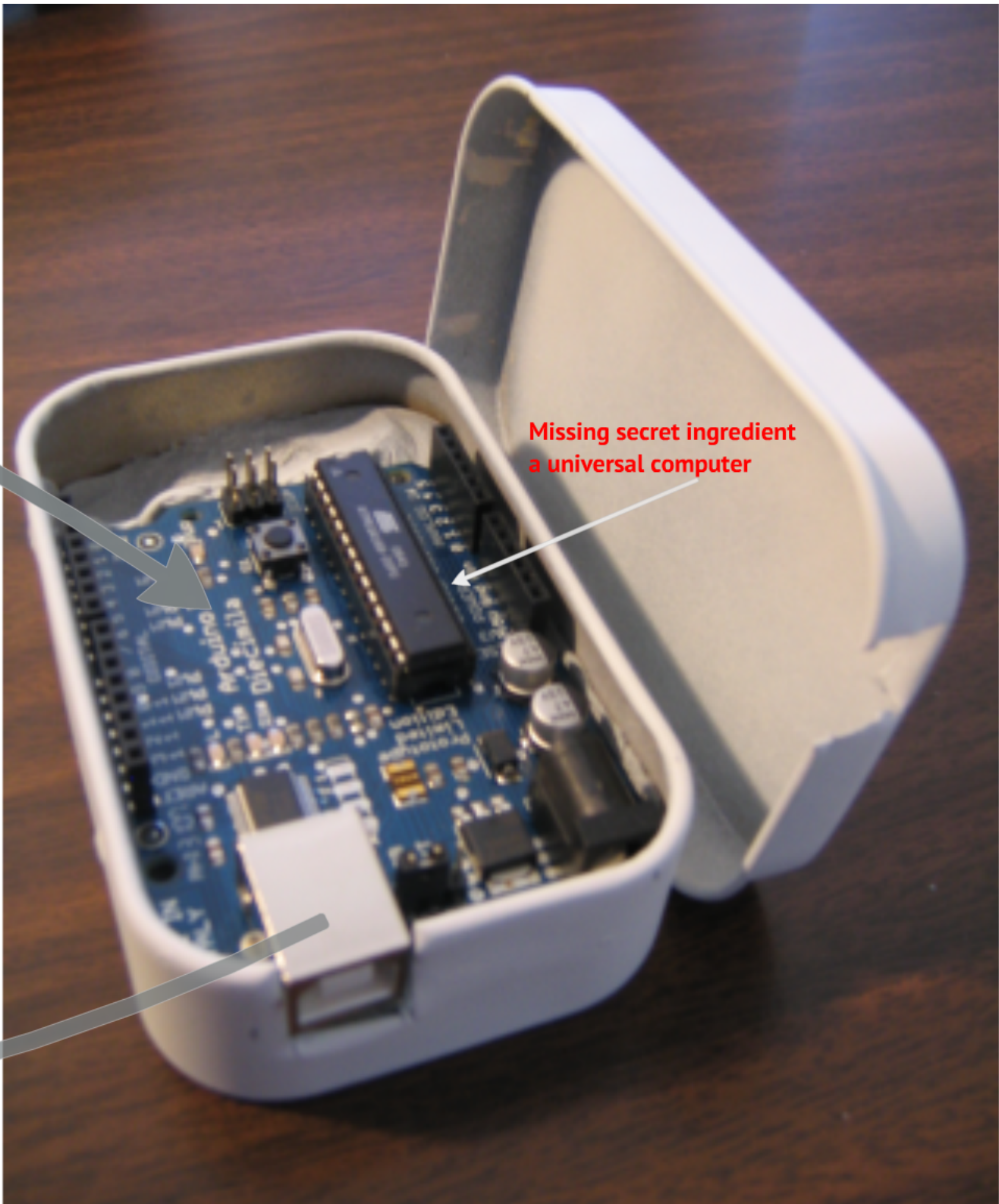
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Missing secret ingredient
a universal computer

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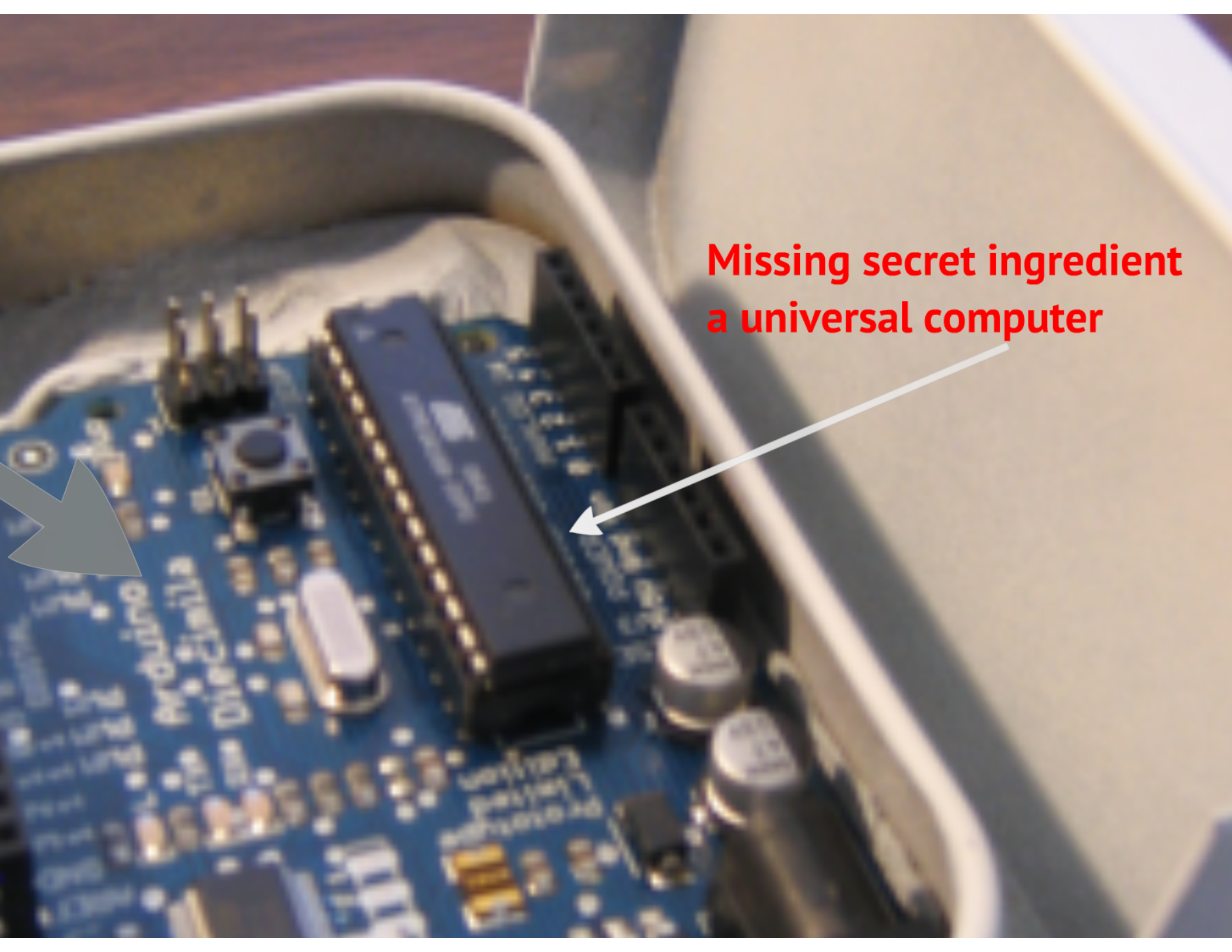
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University
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Greg Michaelson

