An Informational Model for Cellular Automata Aesthetic Measure

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Abstract. This paper addresses aesthetic problem in cellular automata, taking a quantitative approach for aesthetic evaluation. Although the Shannon's entropy is dominant in computational methods of aesthetics, it fails to discriminate accurately structurally different patterns in two-dimensions. We have adapted an informational measure to overcome the shortcomings of entropic measure by using information gain measure. This measure is customised to robustly quantify the complexity of multi-state cellular automata patterns. Experiments are set up with different initial configurations in a two-dimensional multi-state cellular whose corresponding structural measures at global level are analysed. Preliminary outcomes on the resulting automata are promising, as they suggest the possibility of predicting the structural characteristics, symmetry and orientation of cellular automata generated patterns.

1 INTRODUCTION

Cellular Automata (CA) initially invented by von Neumann in the late 1940s as material independent systems to investigate the possibility self-reproduction. His initial cellular automaton to study the possibility of self-reproduction was a two-dimensional (2D) cellular automaton with 29 states and 5-cell neighbourhood. A cellular automaton consists of a lattice of uniformly arranged finite state automata each of which taking input from the neighbouring automata; they in turn compute their next states by utilising a state transition function. A synchronous or asynchronous interactive application of state transition function (also known as a *rule*) over the states of automata (also referred to as *cells*) generates the global behaviour of a cellular automaton.

The formation of complex patterns from simple rules sometimes with high aesthetic quality has been contributed to the creation of many digital art works since the 1960s. The most notable works are "*Pixillation*", one of the early computer generated animations [32], the digital art works of Peter Struycken [31, 36], Paul Brown [5, 12] and evolutionary architecture of John Frazer [18]. Although classical one-dimensional CA with binary states can generate complex behaviours, experiments with 2D multi-state CA have shown that adding more states significantly increases the complexity of behaviour, therefore, generating very complex symmetrical patterns with high aesthetic qualities [21, 22]. These observations have led to the quest of developing a quantitative model to evaluate the aesthetic quality of multi-state CA patterns.

This work follows Birkhoff's tradition in studying mathematical bases of aesthetics, especially the association of aesthetic judgement with the degree of complexity of a stimulus. Shannon's information theory provided an objective measure of complexity. It led to emergence of various informational theories of aesthetics. However due to its nature, the entropic measure fails to take into account spacial characteristics of 2D patterns which is fundamental in addressing aesthetic problem for CA generated patterns.

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2 CELLULAR AUTOMATA ART

The property of CA that makes them particularly interesting to digital artists is their ability to produce interesting and logically deep patterns on the basis of very simply stated preconditions. Iterating the steps of a CA computation can produce fabulously rich output. The significance of CA approach in producing digital art was outlined by Wolfram in his classical studies on CA behaviours in [39]. Traditional scientific intuition, and early computer art, might lead one to assume that simple programs would always produce pictures too simple and rigid to be of artistic interest. But extrapolating from Wolfram's work on CA, "it becomes clear that even a program that may have extremely simple rules will often be able to generate pictures that have striking aesthetic qualities-sometimes reminiscent of nature, but often unlike anything ever seen before" [39, p.11].

Knowlton developed "*Explor*" system for generating 2D patterns, designs and pictures from explicitly provided 2D patterns, local operations and randomness. It aimed not only to provide the computer novice with graphic output; but also a vehicle for depicting results of simulations in natural (i.e. crystal growth) and hypothetical (e.g. cellular automata) situations, and for the production of a wide variety of designs [23]. Together with Schwartz and using *Explor*'s CA models, they generated "*Pixillation*", one of the early computer generated animations [32]. They contested in the *Eighth Annual Computer Art Contest* in 1970 with two entries, "*Tapestry I*" and "*Tapestry II*" (two frames from *Pixillation*). The "*Tapestry I*" won the first prize for "*new, creative use of the computer as an artist's tool*" as noted by selecting committee and covered the front page of *Computers & Automation* on Aug. 1970.

Meertens and Geurts also submitted an entry to the *Eighth Annual Computer Art Contest* with "*Crystalization*" as an experimental computer graphics generated by a asynchronous cellular automaton. Their entries were four drawings intended to generated patterns that combine regularity and irregularity in a natural way [20]. Peter Struycken, the Dutch contemporary digital artist has created many of his works "*Computer Structures*" (1969), "*Four Random Drawings for Lien and Ad*" (1972), "*Fields*" (1979-1980) with binary and multi-state CA [31, 36]. Paul Brown, the British contemporary digital artists also applied various CA rules in his static and kinematic computer arts. "*Neighbourhood Count*" (1991), "*Infinite Permutations V1*" (1993-94), "*Infinite Permutations V2*" (1994-95), "*Sand*

Lines" (1998), "*My Gasket*"(1998) "*Chromos*" (199-2000) [5, 12] are some of his CA generated works.

John F. Simon Jr created a series of art projects called "*Art Appliances*" using a CA based software and LCD panels to exhibit CA pattern formations. "*Every Icon*" (1996), "*ComplexCity*" (2000) and "*Automata Studies*" (2002) are examples of his art appliances. Driessen and Verstappen have produced "*Ima Traveler*" (1996) and "*Breed*"(1995-2007) digital arts in a three-dimensional CA space. Dorin's "*Meniscus*" [13] and McCormack's "*Eden*" [27] are further examples of interactive artworks built on the bases of CA rules. In addition, a combination of CA with other Alife techniques (e.g. evolutionary computing or L-systems) has been used to explore a set of rules generating patterns with aesthetic qualities [9, 34].

Fig. 1 shows some experimental patterns generated by the authors to demonstrate the generative capabilities of CA in creating appealing complex patterns from various initial configurations.

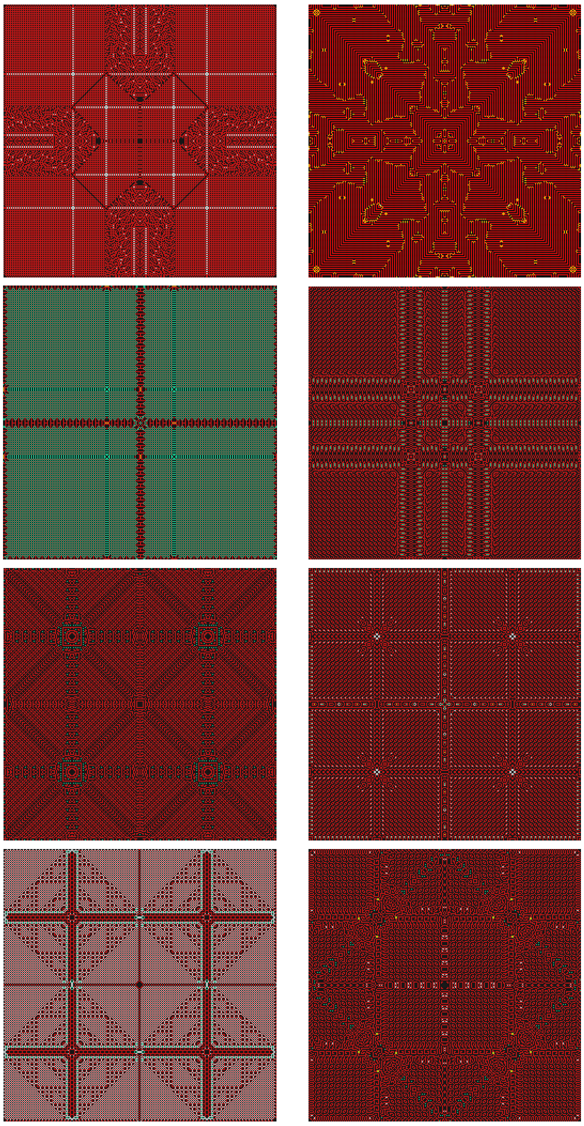


Figure 1. Sample 2D CA generated complex symmetrical patters

3 DEFINITION OF CELLULAR AUTOMATA

In this section, formal notions of 2D CA are explained and later referred to in the rest of the paper.

Definition 1: A cellular automaton is a regular tiling of a lattice with uniform deterministic finite state automata as a quadruple of $\mathcal{A} = \langle L, S, N, f \rangle$ such that:

1. L is an infinite regular *lattice* in $\mathbb{Z}$,
2. $S \subseteq \mathbb{N}^0$ is a finite set of integers as *states*,
3. $N \subseteq \mathbb{N}^+$ is a finite set of integers as *neighbourhood*,
4. $f : S^{|N|} \mapsto S$ is the *state transition function*.

The state transition function f maps from the set of neighbourhood states $S^{|N|}$ where $|N|$ is the cardinality of neighbourhood set, to the set of states $\{s_0, .., s_{n-1}\}$ synchronously in *discrete time* intervals of $t = \{0, 1, 2, 3, ..., n\}$ where t_0 is the *initial time* of a cellular automaton with *initial configuration*. A mapping that satisfies $f(s_0, ..., s_0) = s_0$ where $(s_0 \in S)$, is called a *quiescent state*.

In a 2D square lattice $(\mathbb{Z}^2)$ if the opposite sides of the lattice (up and down with left and right) are connected, the resulting *finite* lattice forms a torus shape (Fig.2) which is referred as a lattice with *periodic boundary conditions*.

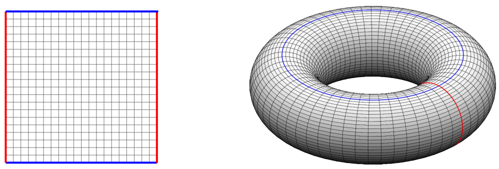


Figure 2. Connecting the opposite sides of a lattice forms a torus

The state of each cell at time $(t+1)$ is determined by the states of immediate surrounding neighbouring cells (nearest neighbourhood) at time (t) given a neighbourhood template. There are two commonly used neighbourhood templates considered for 2D CA. A five-cell mapping $f : S^5 \mapsto S$ known as *von Neumann neighbourhood* (Eq. 1) and a nine-cell mapping $f : S^9 \mapsto S$ known as *Moor neighbourhood* (Eq. 2).

$$s^{t+1}_{i,j} = f \begin{pmatrix} & s^t_{(i,j+1)} & \\ s^t_{(i-1,j)} & s^t_{(i,j)} & s^t_{(i+1,j)} \\ & s^t_{(i,j-1)} & \end{pmatrix} \tag{1}$$

$$s^{t+1}_{i,j} = f \begin{pmatrix} s^t_{(i-1,j+1)} & s^t_{(i,j+1)} & s^t_{(i+1,j+1)} \\ s^t_{(i-1,j)} & s^t_{(i,j)} & s^t_{(i+1,j)} \\ s^t_{(i-1,j-1)} & s^t_{(i,j-1)} & s^t_{(i+1,j-1)} \end{pmatrix} \tag{2}$$

Since the elements of the S are non-negative integers and discrete instances of time are considered, the resulting cellular automaton is a *discrete time-space* cellular automaton. These type of CA can be considered as *discrete dynamical systems*.

4 INFORMATIONAL AESTHETICS

The topic of determining aesthetics or aesthetic measures have been a heated debate for centuries. There is a great variety of computational approaches to aesthetics in visual and auditory forms including mathematical, communicative, structural, psychological and neuroscience. A thorough examination of these methodologies from different perspective has been provided in [19]. In this section, some informational aesthetic measures are presented. Our review is focused on informational theories of aesthetics as these are the ones that conform with this work directly.

Birkhoff suggested an early aesthetic measure by arguing that the measure of aesthetic (M) is in direct relation with the degree of *order* (O) and in reverse relation with the *complexity* (C) of an object [11]. Given that order and complexity are measurable parameters the aesthetic measure of (M) is:

$$M = \frac{O}{C} \tag{3}$$

Even though the validity of Birkhoff's approach to the relationship and definition of order and complexity has been challenged [38, 15, 16, 14], the notion of *complexity* and objective methods to quantify it remains a prominent parameter in aesthetic evaluation functions.

Shannon's introduction of *information theory* provided a mathematical model to measure the degree of uncertainty (entropy) associated with a random variable [33]. The entropy H of a discrete random variable X is a measure of the average amount of uncertainty associated with the value of X. So $H(X)$ as the entropy of X is:

$$H(X) = -\sum_{x \in \mathcal{X}} P(x) \log_2 P(x) \tag{4}$$

The definition of entropy for X has a logarithm in the base of 2 so the unit of measure of entropy is in $bits$.

Moles [28], Bense [7, 6, 8] and Arnheim [2, 3, 4] were pioneers of the application of Shannon's entropy to quantify order and complexity in Birkhoff's formula by adapting statistical measure of information in aesthetic objects. Berlyne used informational approach in his psychological experiments to determine humans perceptual curiosity of visual figures [10]. Bense argued that aesthetic objects are "vehicles of aesthetical information" where statistical information can quantify the aesthetical information of objects [7]. For Bense order is a process of artistic selection of elements from a determined repertoire of elements. The aesthetic measure (M_B) is a the relative redundancy (R) of the reduction of uncertainty because of selecting elements from a repertoire ($H_{max} - H$) to the absolute redundancy (H_{Max}).

$$M_B = \frac{R}{H_{max}} = \frac{H_{max} - H}{H_{max}} \tag{5}$$

where H quantifies entropy of the selection process from a determined repertoire of elements in $bits$ and H_{max} is maximum entropy of predefined repertoire of elements [8]. His informational aesthetics has three basic assumptions. (1) Objects are material carriers of aesthetic state, and such aesthetic states are independent of subjective observers. (2) A particular kind of information is conveyed by the aesthetic state of the object (or process) as *aesthetic information* and (3) objective measure of aesthetic objects is in relation with degree of order and complexity in an object [29].

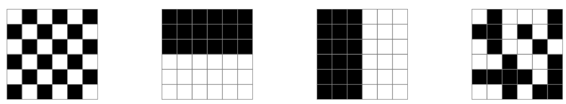
Herbert Franke put forward an *aesthetic perception* theory on the ground of *cybernetic aesthetics*. He made a distinction between the amount of information being stored and the rate of information flowing through a channel as *information flow* measured in $bits/sec$ [17]. His theory is based on psychological experiments which suggested that conscious working memory can not take more than 16 $bits/sec$ of visual information. Then he argued that artists should provide a flow of information of about 16 $bits/sec$ for works of art to be perceived as beautiful and harmonious.

Staudek in his multi criteria approach (informational and structural) as *exact aesthetics* to Birkhoff's measure applied information flow I' by defining it as a measure assessing principal information transmission qualities in time. He used 16 $bits/sec$ reference as channel capacity $C_r = 16\ bits/sec$ and a time reference of 8 seconds ($t_r = 8s$) to argue that artefacts with $I > 128\ bits$ will not fit into the conscious working memory for absorbing the whole aesthetic message [35].

Adapting Bense's informational aesthetics to different approaches of the concepts of order and complexity in an image in [30], three measures based on Kolmogorov complexity [25], Shannon entropy (for RGB channels) and Zurek's physical entropy [40] were introduced. Then the measures were are applied to analyse aesthetic values of several paintings (Mondrian, Pollock, and van Gogh). Machado and Cardoso [26] proposed a model based on Birkhoff's approach as the ratio of *image complexity* to *processing complexity* by arguing that images with high visual complexity, are processed easily so they have highest aesthetic value.

5 INFORMATION GAIN MODEL

Despite the domination of entropic measures to aesthetic evaluation functions, it has a major shortcoming in terms of reflecting structural characteristics of 2D patterns. Examples in Fig.3 illustrate this shortcoming by showing the calculations of entropy for 2D patterns with the same density but different structural regularities and complexities. Fig.3a is a uniformly distributed patterns (a highly ordered pattern), Fig.3b and Fig.3c are patterns with identical structures but in vertical and horizontal orientations. Fig.3d is randomly arranged pattern (a random pattern). As it is evident from the comparison of the patterns and their corresponding entropy value, all of the patterns have the same entropy value. This clearly demonstrates that Shannon's entropy fails to differentiate structural differences among these patterns. In the case of measuring complexity of CA generated patterns especially with multi-state structures, it would be problematic if only entropy used as a measure of complexity for the purpose of aesthetic evaluation.



(a)	(b)	(c)	(d)
$H = 1$	$H = 1$	$H = 1$	$H = 1$

Figure 3. The measure of entropy H for structurally different patterns with the same density of 50%

In order to overcome this problem we have adapted *information gain* model introduced as a method of characterising the complexity of dynamical systems [37]. It has been applied to describe quantitatively the complexity of geometric ornaments and patterns arising in random sequential adsorption of discs on a plane [1]. The information gain G, also known as Kullback-Leibler divergence [24], measures the amount of information required to select a discrete random variable X with state j if prior information about variable X is known at the state of i.

$$G_{x_{ij}} = -\log P_{(x_i|x_j)} \tag{6}$$

where $P_{(x_i|x_j)}$ the conditional probability of the discrete random variable x at state i given its state j. Then from Eq. 6 *mean information gain* $\overline{G}$ would be the average information gain from possible states $(i|j)$:

$$\overline{G} = \sum_{i,j} P(i,j) G_{ij} = -\sum_{i,j} P_{i,j} \log P(i|j) \tag{7}$$

where $P_{(i,j)}$ is the joint probability of the variable x at state i and variable x at state j. Considering Eq. 7, we can define a structural complexity measure for a multi-state 2D cellular automaton as follows:

Definition 2: *A structural complexity measure is the mean information gain of a cell having a heterogeneous neighbouring cell in a multi-state two-dimensional cellular automaton pattern.*

$$\overline{G} = -\sum_{i,j} P_{(i,j)} \log_2 P_{(i|j)} \tag{8}$$

where $P_{(i,j)}$ is the joint probability of a cell having the i state (colour) and the neighbouring cell has the state (colour) j in a given neighbouring cell. And $P_{(i|j)}$ is the conditional probability of the state (colour) i given that its neighbouring cell has state (colour) j in one of four directions of up, low, left or right. The quantity $\overline{G}$ measures average information gain about other elements of the structure (e.g. the state of the neighbouring cell in one of the four directions), when some properties of the structure are known (e.g. the state of a cell). It can be noted that the combined probabilities of $P_{i,j}$ and $P_{i|j}$ describe spatial correlations in a pattern so that $\overline{G}$ can detect inherent correlations of patterns. Considering neighbourhood templates of a 2D CA in Eq.1 and Eq. 2, following variations of $\overline{G}$ can be defined where for each cell in i state given its neighbouring cell in j state in any of directions.

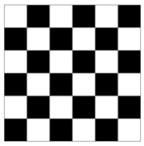
$$\overline{G}_u = -\sum_{i,j_{(x,y+1)}} P_{(i,j_{(x,y+1)})} \log_2 P_{(i|j_{(x,y+1)})} \tag{9}$$

$$\overline{G}_d = -\sum_{i,j_{(x,y-1)}} P_{(i,j_{(x,y-1)})} \log_2 P_{(i|j_{(x,y-1)})} \tag{10}$$

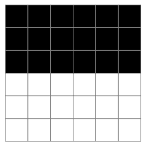
$$\overline{G}_l = -\sum_{i,j_{(x-1,y)}} P_{(i,j_{(x-1,y)})} \log_2 P_{(i|j_{(x-1,y)})} \tag{11}$$

$$\overline{G}_r = -\sum_{i,j_{(x+1,y)}} P_{(i,j_{(x+1,y)})} \log_2 P_{(i|j_{(x+1,y)})} \tag{12}$$

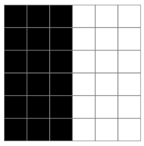
The measure is applied to calculate structural complexly of sample patterns in Fig 4 to demonstrates the ability of $\overline{G}$ in discriminating structurally different 2D patterns. The calculations have been performed for each elements of the patterns having a heterogamous colour in one of the four directions from two possible colours.



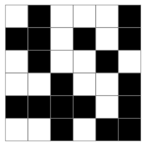
(a)
$\overline{G}_u = 0$
$\overline{G}_d = 0$
$\overline{G}_l = 0$
$\overline{G}_r = 0$
$H = 1$



(b)
$\overline{G}_u = 0.5510$
$\overline{G}_d = 0.5510$
$\overline{G}_l = 0$
$\overline{G}_r = 0$
$H = 1$



(c)
$\overline{G}_u = 0$
$\overline{G}_d = 0$
$\overline{G}_l = 0.5510$
$\overline{G}_r = 0.5510$
$H = 1$



(d)
$\overline{G}_u = 0.9839$
$\overline{G}_d = 0.9871$
$\overline{G}_l = 0.9377$
$\overline{G}_r = 0.9473$
$H = 1$

Figure 4. The comparison of entropy H and $\overline{G}$ for structurally different patterns but with the same density of 50%

6 EXPERIMENTS AND RESULTS

A set of experiments were designed to examine the effectiveness of $\overline{G}$ in discriminating structurally different patterns generated by multi-state 2D CA. The chosen experimental cellular automaton maps three states represented by *green*, *red* and *blue* colour cells. The quiescent state cells represented with *white* colours. The size of the lattice is set to 129×129 cells. Two set of experiments are conducted: (1) a single cell as initial configuration and (2) a randomly seeded initial configuration with 50% destiny of three states (*green*, *red*, *blue*). Both of the experiments are conducted for 300 successive time steps. The $\overline{G}$ for four directions along with their corresponding entropy H are measured in $bits$.

Fig. 7 and Fig. 8 illustrate the formation of 2D patterns for a sample of 12 time steps $\{0, 10, 20, 30, 40, 50, 60, 70, 80, 100, 200, 300\}$ starting from two different initial configurations and their corresponding $\overline{G}$ and H. Figs. 7 and 6 shows the plots of $\overline{G}$ and H for 300 time steps. The $\overline{G}$ measures in Fig. 7 which shows the formation of 2D patterns from a single cell are conforming in directional calculations; it means that each cell in the patterns has exactly the same amount of information regarding their neighbouring cell in one of the four directions. Therefore it indicates that the development of the patterns are symmetrical in the four directions. In other words, the cellular automaton with a single cell as its initial configuration has created 2D pattens with four-fold rotational symmetry. The measure in Fig. 8 starts with $\overline{G} \approx 1.7$ for the random initial configuration and with $H \approx 1.5$ (maximum entropy for a three-state patterns since $\log_2 3 = 1.5848$). The formation of patterns with local structures reduces the value of $\overline{G}$. The values of $\overline{G}$ are not conforming in any directional calculations which indicates the development of less ordered ("chaotic") patterns. From the comparison of H with $\overline{G}$ in the set of experiments, it is clear that it would be very unlikely to discriminate the structural differences of patterns with a single measure

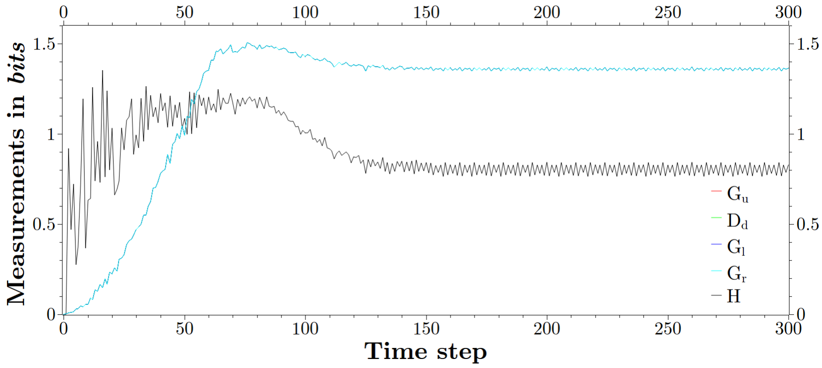


Figure 5. The plot of $\overline{G}$ and H for 300 time steps starting from single cell

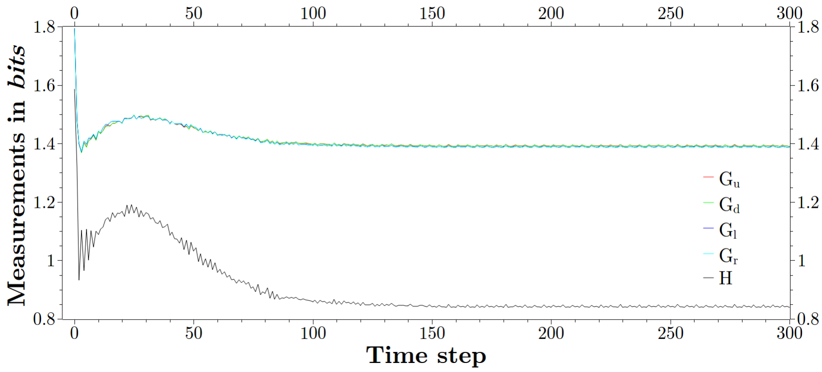


Figure 6. The plot of $\overline{G}$ and H for 300 time steps starting from random initial configuration

of H given the diversity of patterns that can be generated by various 2D CA state transition functions. Computing directional measures of $\overline{G}$ and comparing their values provides a more subtle measure of structural order and complexity of a 2D pattern. The conformity or non-conformity of $\overline{G}$ measure in up, down, left and right neighbouring cells clearly gives us not only an accurate measure of structural characteristics of 2D patterns but they also provide us with information about the orientation of the patterns as well.

7 CONCLUSION

Cellular automata (CA), which are fundamental to the study of self-replicating systems, are powerful tools in generating computer art. The multi-state 2D CA rule space is a vast set of possible rules which can generate interesting patterns with high aesthetic qualities. The application of CA in digital art has been reviewed; and the concepts of order and complexity from Shannon's information entropy perspective in the CA framework has been analysed concluding that existing informational aesthetic measures do not capture structural differences in 2D patterns. In order to address the shortcomings of informational approaches to computational aesthetics, a mean information gain model was adapted to measure both structural complexity and distinguish symmetrical orientation of 2D CA patterns. The measure takes into account conditional and joint probabilities of the information gain value that a cell offers, given a particular position of its neighbouring cells. The effectiveness of the measure is shown in a series of experiments for multi-state 2D patterns generated by a cellular automaton. The results of the experiments show that the mean information gain model is capable of distinguishing the structural complexity of 2D CA patterns as well as their symmetrical orientation. Having a model to evaluate the aesthetic qualities of CA generated patterns could potentially have a substantial contribution towards further automation of the evaluative component in the CA based computer generated art. This could also enable us to have an integrated process of generation-evaluation which is a subject of on going research.

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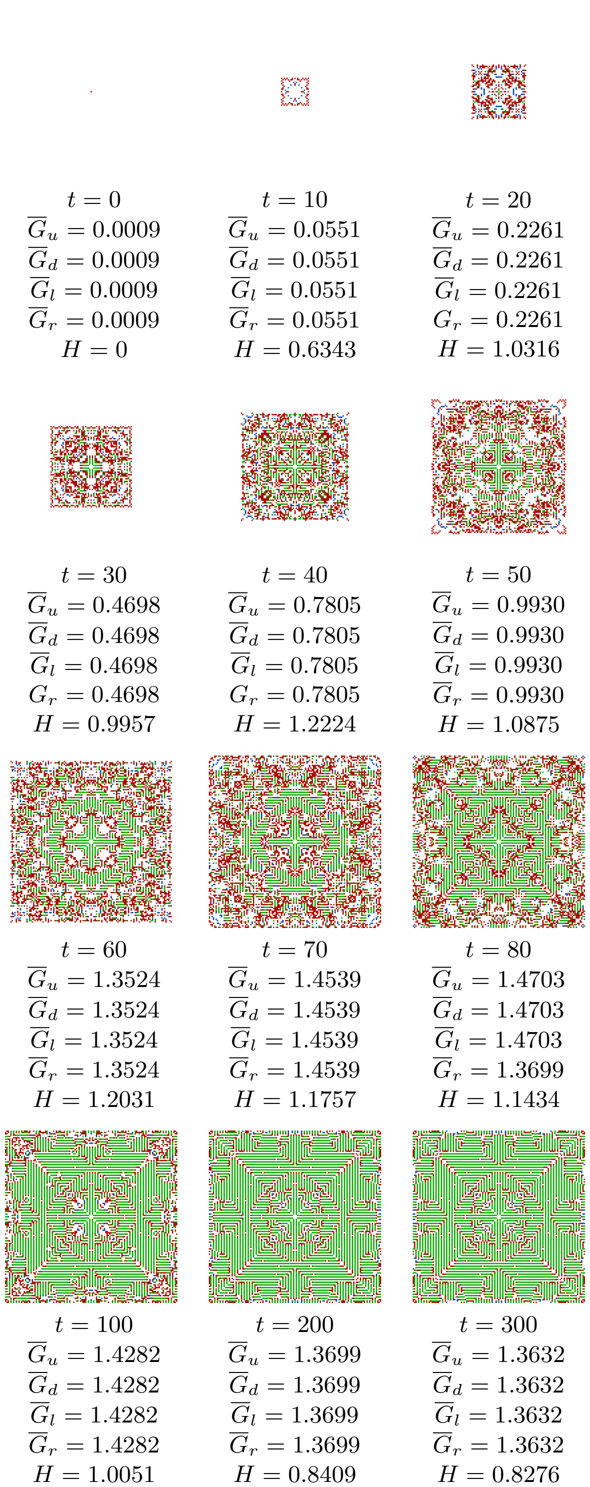

Figure 7. Patterns generated from a single cell as initial configuration and their corresponding $\overline{G}$ and H values

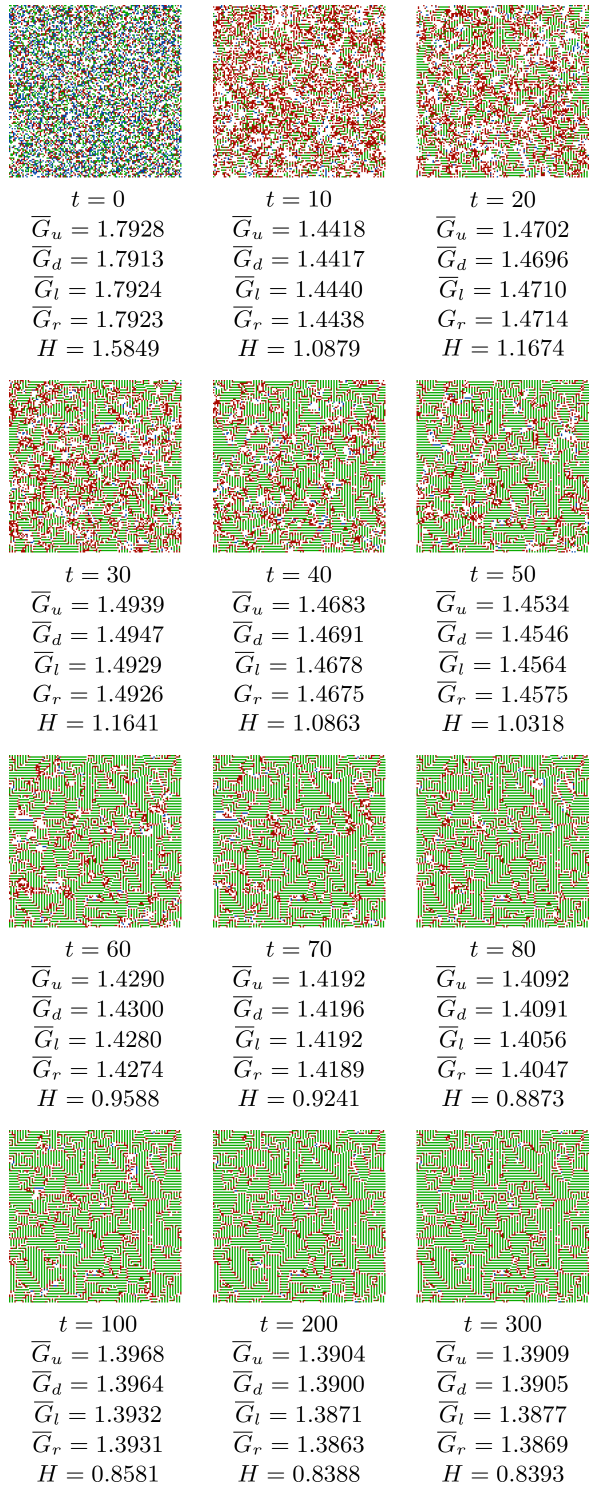

Figure 8. Patterns generated from a 50% seeded density as initial configuration and their corresponding $\overline{G}$ and H values

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