Collapsing Closures

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Outline of this talk

The set-sharing domain
- Descriptions of substitutions
- Descriptions of interpretations

The computational problem with set-sharing
- Abstract unification
- Bindings imposed by body atoms
- Projection
- Deferring closure evaluation

Repairing set-sharing
- Tagging sharing groups with identifiers
- Rules for collapsing closures
- Experimental results

Conclusions

Familiar program

append(Xs,Ys,Zs) :- Xs = [], Ys = Zs.
append(Xs,Ys,Zs) :- Xs = [X|Vs], Zs = [X|Ws], append(Vs,Ys,Ws).
The behaviour can be captured with a T-operator:

$$
\begin{array}{ll}
I_0 = \emptyset & \\
I_1 = \{\mathrm{append}(Xs, Ys, Zs) :\!- \theta_1\} & \text{where } \theta_1 = \{Xs \mapsto [], Ys \mapsto Zs\} \\
I_2 = \{\mathrm{append}(Xs, Ys, Zs) :\!- \theta_2\} \cup I_1 & \text{where } \theta_2 = \{Xs \mapsto [X], Zs \mapsto [X|Ys]\} \\
I_3 = \{\mathrm{append}(Xs, Ys, Zs) :\!- \theta_3\} \cup I_2 & \text{where} \\
 & \theta_3 = \{Xs \mapsto [X, Y], Zs \mapsto [X, Y|Ys]\} \\
I_i = \{\mathrm{append}(Xs, Ys, Zs) :\!- \theta_i\} \cup I_{i-1} & \text{where} \\
\multicolumn{2}{r}{\theta_i = \{Xs \mapsto [X_1, \ldots, X_{i-1}], Zs \mapsto [X_1, \ldots, X_{i-1}|Ys]\}}
\end{array}
$$

The limit of the sequence (lfp of T operator) is the interpretation $I = \{\mathrm{append}(Xs, Ys, Zs) :\!- \theta_i \mid i \in \mathbb{N}\}$ which is an *infinite* set.

A set-sharing description is a *set* of sharing groups

- For $\theta_3 = \{Xs \mapsto [X, Y], Zs \mapsto [X, Y|Ys]\}$ it follows that:

$$\begin{aligned} occ(\theta_3, X\ ) &= \{Xs, Zs, X\} \\ occ(\theta_3, Y\ ) &= \{Xs, Zs, Y\} \\ occ(\theta_3, Xs) &= \emptyset \\ occ(\theta_3, Ys) &= \{Ys, Zs\} \\ occ(\theta_3, Zs) &= \emptyset \\ occ(\theta_3, y\ ) &= \{y\} \quad \text{where} \quad y \notin \{Xs, Ys, Zs, X, Y\} \end{aligned}$$

- The abstraction map $\alpha_{\mathcal{X}}(\theta) = \{occ(\theta, y) \cap \mathcal{X} \mid y \in \mathcal{V}\}$ for a *single* substitution parameterised by a *finite* set of variables $\mathcal{X}$.
- If $\mathcal{X} = \{Xs, Ys, Zs\}$ then $\alpha_{\mathcal{X}}(\theta_3) = \{\emptyset, \{Ys, Zs\}, \{Xs, Zs\}\}$.
- The *description* $\alpha_{\mathcal{X}}(\theta_3)$ shows that Xs and Ys do not share

J_i as a description of I_i

- The abstraction map $\alpha_{\mathcal{X}}(\theta)$ for a single θ is sufficient to judge the correctness of an abstract version of the T operator
- Successive interpretations J_i are deemed to be correct iff for each constrained atom append(Xs, Ys, Zs) :- $\theta \in I_i$ there exists append(Xs, Ys, Zs) :- $S \in J_i$ such that $\alpha_{\mathcal{X}}(\theta) \subseteq S$.

$$
\begin{aligned}
J_0 &= \emptyset \\
J_1 &= \{\text{append}(Xs, Ys, Zs) :\!-\, S'_1\} \qquad \text{where } S'_1 = \{\emptyset, \{Ys, Zs\}\} \\
J_2 &= \{\text{append}(Xs, Ys, Zs) :\!-\, S'_2\} \cup J_1 \text{ where} \\
&\qquad S'_2 = \{\emptyset, \{Xs, Zs\}, \{Ys, Zs\}, \{Xs, Ys, Zs\}\} \\
J_3 &= J_2
\end{aligned}
$$

Overview of computing J_3 with second clause in 5 steps

append(Xs,Ys,Zs) :- Xs = [X|Vs], Zs = [X|Ws], append(Vs,Ys,Ws)

.

1. Set initial description to $S_0 = \alpha_{\mathcal{X}}(\varepsilon) = \{\emptyset, \{Vs\}, \{Ws\}, \{X\}, \{Xs\}, \{Ys\}, \{Zs\}\}$ where $\mathcal{X} = \{Vs, Ws, X, Xs, Ys, Zs\}$
2. Simulate Xs = [X|Vs] with input S_0 to obtain output S_1:
   - apply $S_1 = amgu(Xs, [X|Vs], S_0)$
   - comes with the correctness guarantee that if $\alpha_{\mathcal{X}}(\theta) \subseteq S_0$ and $\delta \in mgu(\theta(Xs), \theta([X|Vs]))$ then $\alpha_{\mathcal{X}}(\delta \circ \theta) \subseteq S_1$
3. Apply $S_2 = amgu(Zs, [X|Ws], S_1)$
4. Add the bindings imposed append(Vs, Ys, Ws) to S_2 to obtain S_9 (amounts to several operations)
5. Restrict S_9 to variable set $\{Xs, Ys, Zs\}$ to obtain finally obtain the description $S_{10} = \{\emptyset, \{Xs, Zs\}, \{Ys, Zs\}, \{Xs, Ys, Zs\}\}$

The problem at the heart of the *amgu* (a bit slower)

- Put $S_0 = \{\emptyset, \{Vs\}, \{Ws\}, \{X\}, \{Xs\}, \{Ys\}, \{Zs\}\}$ (step 1)
- Compute $S_1 = amgu(X, [X|Vs], S_0)$ where:

$$amgu(X, [X|Vs], S_0) = S_0 \setminus (T_1 \cup T_2) \cup cl(T_1 \uplus T_2)$$

$$\begin{aligned} T_1 = rel(Xs, S_0) &= \{\{Xs\}\} \\ T_2 = rel([X|Vs], S_0) &= \{\{Vs\}, \{X\}\} \\ T_1 \uplus T_2 &= \{\{Vs, Xs\}, \{X, Xs\}\} \\ cl(T_1 \uplus T_2) &= \{\{Vs, Xs\}, \{Vs, X, Xs\}, \{X, Xs\}\} \end{aligned}$$

  to obtain $S_1 = \{\emptyset, \{Vs, Xs\}, \{Vs, X, Xs\}, \{Ws\}, \{X, Xs\}, \{Ys\}, \{Zs\}\}$ (step 2)
- Computing $S_2 = amgu(Zs, [X|Ws], S_1)$ is analogous (step 3)

The problem with closure made worse (step 4)

- The bindings imposed by the body atom append(Vs, Ys, Ws) need to be added to S_2.
- These bindings are recorded in J_2, not in terms of append(Vs, Ys, Ws), but in terms of append(Xs, Ys, Zs).
- Extend S_2 to give $S_3 = S_2 \cup \{\{\underline{Xs}, \underline{Zs}\}, \{\underline{Ys}, \underline{Zs}\}, \{\underline{Xs}, \underline{Ys}, \underline{Zs}\}\}$ where $\underline{Xs}$, $\underline{Ys}$ and $\underline{Zs}$ are fresh variables.
- Then interleave *amgu* and projection like so:
  - $S_4 = amgu(Vs, \underline{Xs}, S_3)$ and $S_5 = S_4 \upharpoonright (\mathcal{X} \setminus \{\underline{Xs}\})$
  - $S_6 = amgu(Ys, \underline{Ys}, S_5)$ and $S_7 = S_6 \upharpoonright (\mathcal{X} \setminus \{\underline{Ys}\})$
  - $S_8 = amgu(Ws, \underline{Zs}, S_7)$ and $S_9 = S_8 \upharpoonright (\mathcal{X} \setminus \{\underline{Zs}\})$

Intermediate variables aggravate closure calculation

$$
\begin{aligned}
S_4 &= \{\emptyset, \{\underline{Xs}, \underline{Ys}, \underline{Zs}, Vs, Ws, X, Xs, Zs\}, \{\underline{Xs}, \underline{Ys}, \underline{Zs}, Vs, X, Xs, Zs\}, \\
&\quad \{\underline{Xs}, \underline{Ys}, \underline{Zs}, Vs, Xs\}, \{\underline{Xs}, \underline{Zs}, Vs, Ws, X, Xs, Zs\}, \\
&\quad \{\underline{Xs}, \underline{Zs}, Vs, X, Xs, Zs\}, \{\underline{Xs}, \underline{Zs}, Vs, Xs\}, \{\underline{Ys}, \underline{Zs}\}, \\
&\quad \{Ws, X, Xs, Zs\}, \{Ws, Zs\}, \{X, Xs, Zs\}, \{Ys\}\} \\
S_5 &= \{\emptyset, \{\underline{Ys}, \underline{Zs}, Vs, Ws, X, Xs, Zs\}, \{\underline{Ys}, \underline{Zs}, Vs, X, Xs, Zs\}, \\
&\quad \{\underline{Ys}, \underline{Zs}, Vs, xs\}, \{\underline{Zs}, Vs, Ws, X, Xs, Zs\} \\
&\quad \{\underline{Zs}, Vs, X, Xs, Zs\}, \{\underline{Zs}, Vs, Xs\}, \{\underline{Ys}, \underline{Zs}\}, \\
&\quad \{Ws, X, Xs, Zs\}, \{Ws, Zs\}, \{X, Xs, Zs\}, \{Ys\}\} \\
&\ \vdots \\
S_9 &= \{\emptyset, \{Vs, Ws, X, Xs, Ys, Zs\}, \{Vs, Ws, Xs, Ys, Zs\}, \\
&\quad \{Vs, Ws, X, Xs, Zs\}, \{Vs, Ws, Xs, Zs\}, \{Ws, X, Xs, Ys, Zs\}, \\
&\quad \{Ws, Ys, Zs\}, \{X, Xs, Zs\}\}
\end{aligned}
$$

Recovering tractability with projection (step 5)

- The description $S_9 = \{\emptyset, \{Vs, Ws, X, Xs, Ys, Zs\}, \{Vs, Ws, Xs, Ys, Zs\}, \{Vs, Ws, X, Xs, Zs\}, \{Vs, Ws, Xs, Zs\}, \{Ws, X, Xs, Ys, Zs\}, \{Ws, Ys, Zs\}, \{X, Xs, Zs\}\}$ expresses the bindings imposed on the variables of the *whole clause*.
- The restriction $S_{10} = S_9 \upharpoonright \{Xs, Ys, Zs\} = \{\emptyset, \{Xs, Zs\}, \{Ys, Zs\}, \{Xs, Ys, Zs\}\}$ then describes bindings on the head.
- Since $S_{10} = S_2$ (original) it follows that a fixpoint has been reached and therefore J_2 describes the limit I.

Observation: this application of the abstract T requires 5 closures that all compute complicated descriptions over ≥ 6 variables.
Irony: S_{10} is simple since it is defined over 3 variables.

Idea: defer evaluation until a propitious moment

- Consider again the definition

$$amgu(t_1, t_2, S) = (S \setminus (T_1 \cup T_2)) \cup cl(T_1 \uplus T_2)$$

- Instead of computing $cl(T_1 \uplus T_2)$, the tactic is to tag all the groups within $T_1 \uplus T_2$ with an identifier — a unique number — that identifies those groups that participate in the closure.
- The tags are retained until head projection whereupon they are used to activate closure calculation.
- Then the tags are discarded.

Tagging with an identifier (new approach to step 2)

Put $S_0 = \{\emptyset, \{Vs\}, \{Ws\}, \{X\}, \{Xs\}, \{Ys\}, \{Zs\}\}$ (step 1)
Compute $S_1 = amgu'(X, [X|Vs], 1, S_0)$ where:

$$amgu'(t_1, t_2, n, S) = (S \setminus (T_1 \cup T_2)) \cup tag(T_1 \uplus' T_2, n)$$

$$\begin{aligned} T_1 = rel(Xs, S_0) &= \{\{Xs\}\} \\ T_2 = rel([X|Vs], S_0) &= \{\{Vs\}, \{X\}\} \\ T_1 \uplus T_2 &= \{\{Vs, Xs\}, \{X, Xs\}\} \\ tag(T_1 \uplus T_2, 1) &= \{\langle\{Vs, Xs\}, \{1\}\rangle, \langle\{X, Xs\}, \{1\}\rangle\} \end{aligned}$$

hence $S_1 = \{\emptyset, \langle\{Vs, Xs\}, \{1\}\rangle, \{Ws\}, \langle\{X, Xs\}, \{1\}\rangle, \{Ys\}, \{Zs\}\}$

Tagging with two identifiers (new approach to step 3)

Repeating this tactic for $S_2 = amgu'(Zs, [X|Ws], 2, S_1)$ yields:

$$\begin{array}{rcl} T_1 = rel(Zs, S_1) & = & \{\{Zs\}\} \\ T_2 = rel([X|Ws], S_1) & = & \{\{Ws\}, \langle\{X, Xs\}, \{1\}\rangle\} \\ T_1 \uplus T_2 & = & \{\{Ws, Zs\}, \langle\{X, Xs, Zs\}, \{1\}\rangle\} \\ tag(T_1 \uplus T_2, 2) & = & \{\langle\{Ws, Zs\}, \{2\}\rangle, \langle\{X, Xs, Zs\}, \{1, 2\}\rangle\} \end{array}$$

$$\text{and } S_2 = \{\emptyset, \langle\{Vs, Xs\}, \{1\}\rangle, \langle\{Ws, Zs\}, \{2\}\rangle, \\ \langle\{X, Xs, Zs\}, \{1, 2\}\rangle, \{Ys\}\}$$

Identifiers 1 and 2 show which groups participate in which closures.

Rules for collapsing closures (after head projection)

- Duplicated group rule:
  - $S_{10} = \{\emptyset, \langle\{\mathbf{Xs}, \mathbf{Zs}\}, \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{5}\}\rangle, \langle\{Xs, Zs\}, \{1, 2, 3, 4, 5\}\rangle, \langle\{Ys, Zs\}, \{2, 4, 5\}\rangle, \langle\{\mathbf{Xs}, \mathbf{Zs}\}, \{\mathbf{1}, \mathbf{2}\}\rangle\}$
  - $S_{11} = \{\emptyset, \langle\{\mathbf{Xs}, \mathbf{Zs}\}, \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{5}\}\rangle, \langle\{\mathbf{Xs}, \mathbf{Zs}\}, \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}\}\rangle, \langle\{Ys, Zs\}, \{2, 4, 5\}\rangle\}$
  - $S_{12} = \{\emptyset, \langle\{Xs, Zs\}, \{1, 2, 3, 4, 5\}\rangle, \langle\{Ys, Zs\}, \{2, 4, 5\}\rangle\}$
- Uniqueness rule:
  - $S_{12} = \{\emptyset, \langle\{Xs, Zs\}, \{\mathbf{1}, \mathbf{2}, 3, 4, 5\}\rangle, \langle\{Ys, Zs\}, \{2, 4, 5\}\rangle\}$
  - $S_{13} = \{\emptyset, \langle\{Xs, Zs\}, \{2, 4, 5\}\rangle, \langle\{Ys, Zs\}, \{2, 4, 5\}\rangle\}$
- Covering rule:
  - $S_{13} = \{\emptyset, \langle\{Xs, Zs\}, \{\mathbf{2}, \mathbf{4}, 5\}\rangle, \langle\{Ys, Zs\}, \{\mathbf{2}, \mathbf{4}, 5\}\rangle\}$
  - $S_{14} = \{\emptyset, \langle\{Xs, Zs\}, \{\mathbf{4}, \mathbf{5}\}\rangle, \langle\{Ys, Zs\}, \{\mathbf{4}, \mathbf{5}\}\rangle\}$
  - $S_{15} = \{\emptyset, \langle\{Xs, Zs\}, \{5\}\rangle, \langle\{Ys, Zs\}, \{5\}\rangle\}$

Experimental results for goal-dependent/goal-independent

	collapsed		classic		collapsed		classic	
	time	clos'es	time	clos'es	time	clos'es	time	clos'es
ann	2734	200	11661	806	5564	615	5812	2916
asm	172	247	140	563	937	500	20701	2299
conman	1187	32	1235	136	1906	93	1813	326
crip	438	132	6766	946	5093	560	25483	3917
cs_r	2687	149	–	–	516	32	3250	204
disj_r	110	45	8500	321	219	16	94	105
ga	203	47	672	141	1422	105	11966	348
life	516	33	547	114	1532	18	1500	77
nbody	938	93	7344	267	5641	489	–	–
peep	125	333	329	619	5077	1576	17174	3752
press	266	293	641	801	1937	1184	6156	4137

Conclusions

- Paper includes “bonus tracks” which establish correctness;
- Suspect that closure collapsing combines with:
  - groundness dependencies
  - linearity
  - freeness
- Investigate how closure collapsing can be applied with:
  - and-or tree framework [Bruynooghe, JLP, 1991]
  - induced-magic framework [Codish, JLP, 1999]
  - early projection [folklore]
  - Boolean function encoding as a correctness abstraction [Codish, Søndergaard, Stuckey, TOPLAS, 1999]
- Suggests the general technique of procrastination