On Modular-Reduction Vulnerabilities

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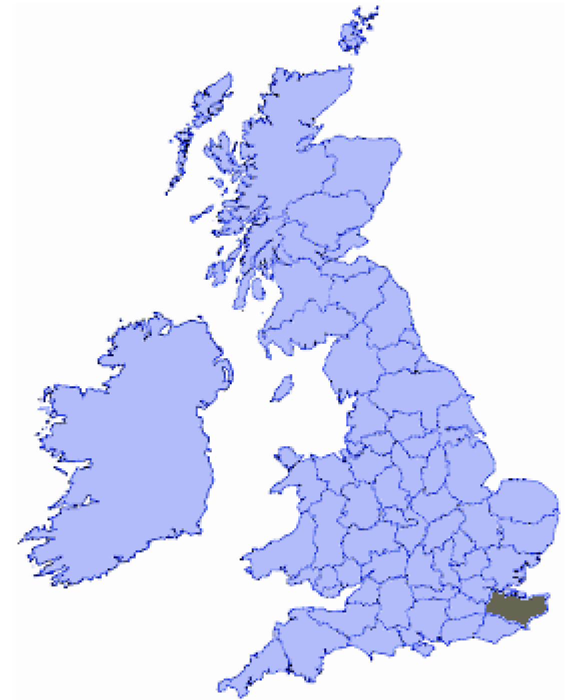
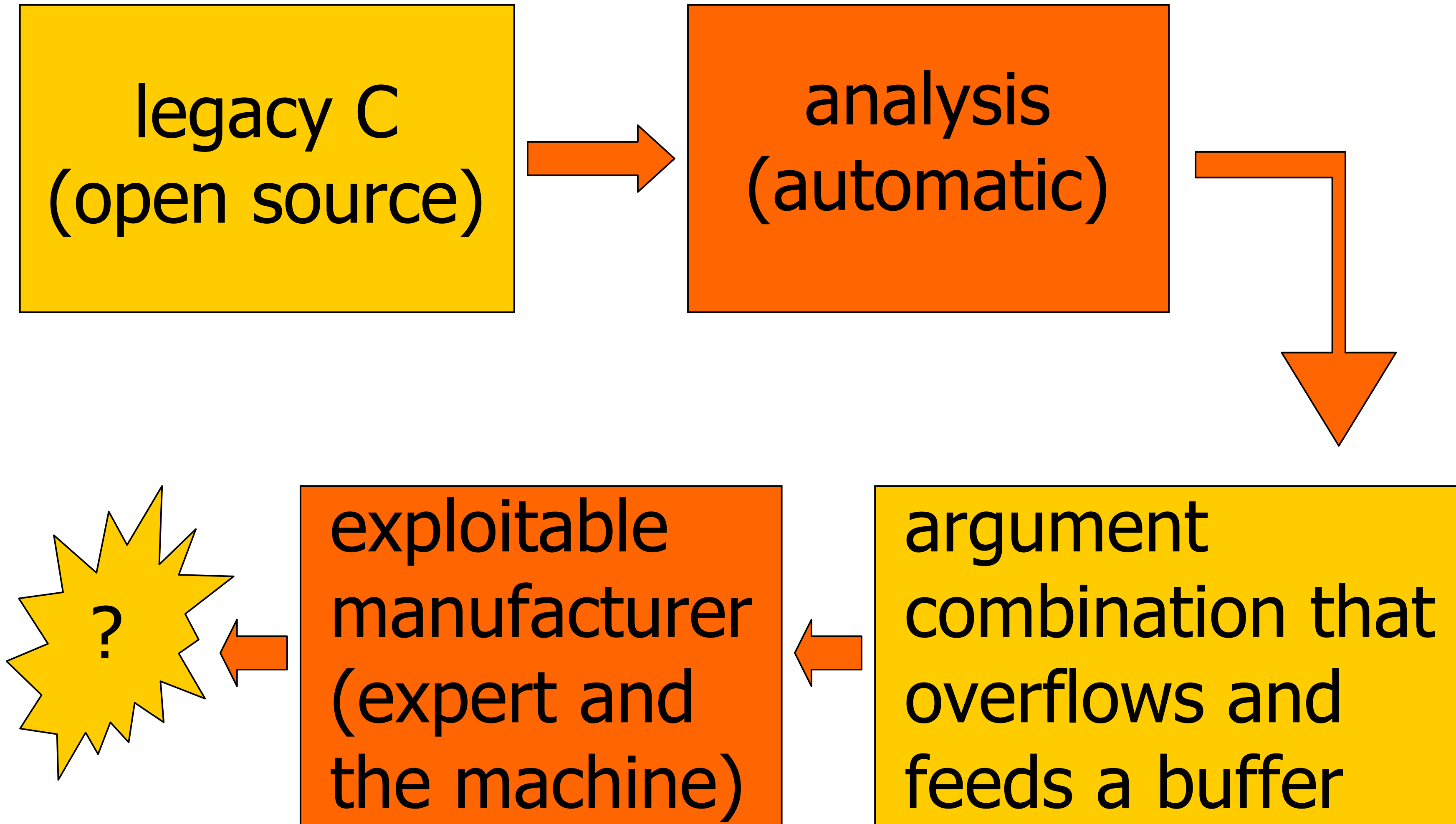


Our agenda



deattack.c of OpenSSH version 2.3.0 (1 of 2)

```
#define SSH_MAXBLOCKS    (32 * 1024)
#define SSH_BLOCKSIZE    (8)
...
#define HASH_MINSIZE     (8 * 1024)
#define HASH_ENTRYSIZE   (2)
#define HASH_FACTOR(x)   ((x) * 3/2)
...
int
detect_attack(unsigned char *buf, u_int32_t len,
              unsigned char *IV)
{   static u_int16_t *h = (u_int16_t *) NULL;
    static u_int16_t n = HASH_MINSIZE / HASH_ENTRYSIZE;
    register u_int32_t i, j;
    u_int32_t l;
    ...
```

←remember (annotation on the `HASH_FACTOR(x)` line)

← 87387 ≤ len ≤ 262144 and len is a multiple of 8 (annotation on the `int` line)

↑n = (8 * 1024) / 2 = 4096 (annotation on the `static u_int16_t n` line)

deattack.c of OpenSSH version 2.3.0 (2 of 2)

```
u_int32_t l;
... ↓len ≤ 262144 = (32 * 1024) * 8
if (len > (SSH_MAXBLOCKS * SSH_BLOCKSIZE) ||
    len % SSH_BLOCKSIZE != 0)  ←len % 8 = 0
{  fatal("detect_attack: bad length %d", len);   }
for (l = n; l < HASH_FACTOR(len / SSH_BLOCKSIZE); l = l << 2)
   ; ↑l = 4096 =2^12   ↑2^14+1=16385=3(87387)/16≤3(len/8)/2
if (h == NULL) {
   debug("Installing crc compensation attack detector.");
   n = l;          ←n = l = 2^16 or n = l = 2^18 or ...
   h = (u_int16_t *) xmalloc(n * HASH_ENTRYSIZE);
} else {                              ↑0 * 2 = 0
   if (l > n) {
      n = l;
      h = (u_int16_t *) xrealloc(h, n * HASH_ENTRYSIZE);
   }                                          ↑0 * 2 = 0
}
}
```

OpenSSH is the "first-fruits" of a new class of vulnerability

- So when $87387 \le len \le 262144$ and len is a multiple of 8 then, due to 16 bit overflow, a buffer overflow occurs
- Similar vulnerabilities now known to exist in
  - glibc 2.1.3
  - OpenBSD select()
  - Apache chunked encoding memcpy()
- But integer overflows are silent and ubiquitous; these are the *known* ones

Existing approaches (circa 2004)

- Decision procedures such as Fourier-Motzkin:
  - assume arbitrary precision
  - do not support logical and bit-ops like || and <<
- Congruence domains [Granger, SAS, 1997; Müller-Olm, ESOP, 2005]
  - do not support bit-ops
- Automatic test data generation [Offutt, SPE, 1999; Gotlieb, CL, 2000]:
  - model word operations with finite domain constraints
  - use reification to *enforce* overflow *somewhere*
  - solve system and then project onto arguments
  - need 16- and 32-bit objects but SICStus imposes a painful 30-bit restriction

Tying down a taint (1 of 3)

$$\langle x = \langle b_1,\ldots,b_w\rangle, \sigma\rangle \to \langle \text{skip}, \sigma[\langle b_1,\ldots,b_w,0\rangle/x]\rangle$$

$$\frac{\vee_{i=1}^{w} \sigma(x)^i = 1}{\langle \text{if } x \text{ then } s_1 \text{ else } s_2, \sigma\rangle \to \langle s_1, \sigma\rangle}$$

$$\frac{\vee_{i=1}^{w} \sigma(x)^i = 0}{\langle \text{if } x \text{ then } s_1 \text{ else } s_2, \sigma\rangle \to \langle s_2, \sigma\rangle}$$

Tying down a taint (2 of 3)

$$b_i = \sigma(y)^i \wedge \sigma(z)^i \quad \text{for all } \; i \in [1,w]$$
$$b_{w+1} = \sigma(y)^{w+1} \vee \sigma(z)^{w+1}$$
$$\overline{\langle x = y \;\&\; z, \sigma\rangle \rightarrow \langle \text{skip}, \sigma[\langle b_1,\ldots,b_w,b_{w+1}\rangle/x]\rangle}$$

Tying down a taint (3 of 3)

$$c_1 = 0$$

$$c_{i+1} = (\sigma(y)^i \wedge \sigma(z)^i) \vee (\sigma(y)^i \wedge c_i) \vee (\sigma(z)^i \wedge c_i)$$

for all $i \in [1,w]$

$$b_i = \sigma(y)^i \oplus \sigma(z)^i \oplus c_i \quad \text{for all } i \in [1,w]$$

$$b_{w+1} = \sigma(y)^{w+1} \vee \sigma(z)^{w+1} \vee c_{w+1}$$

$$\langle x = y + z, \sigma \rangle \rightarrow \langle \text{skip}, \sigma[\langle b_1, \ldots, b_w, b_{w+1} \rangle / x] \rangle$$

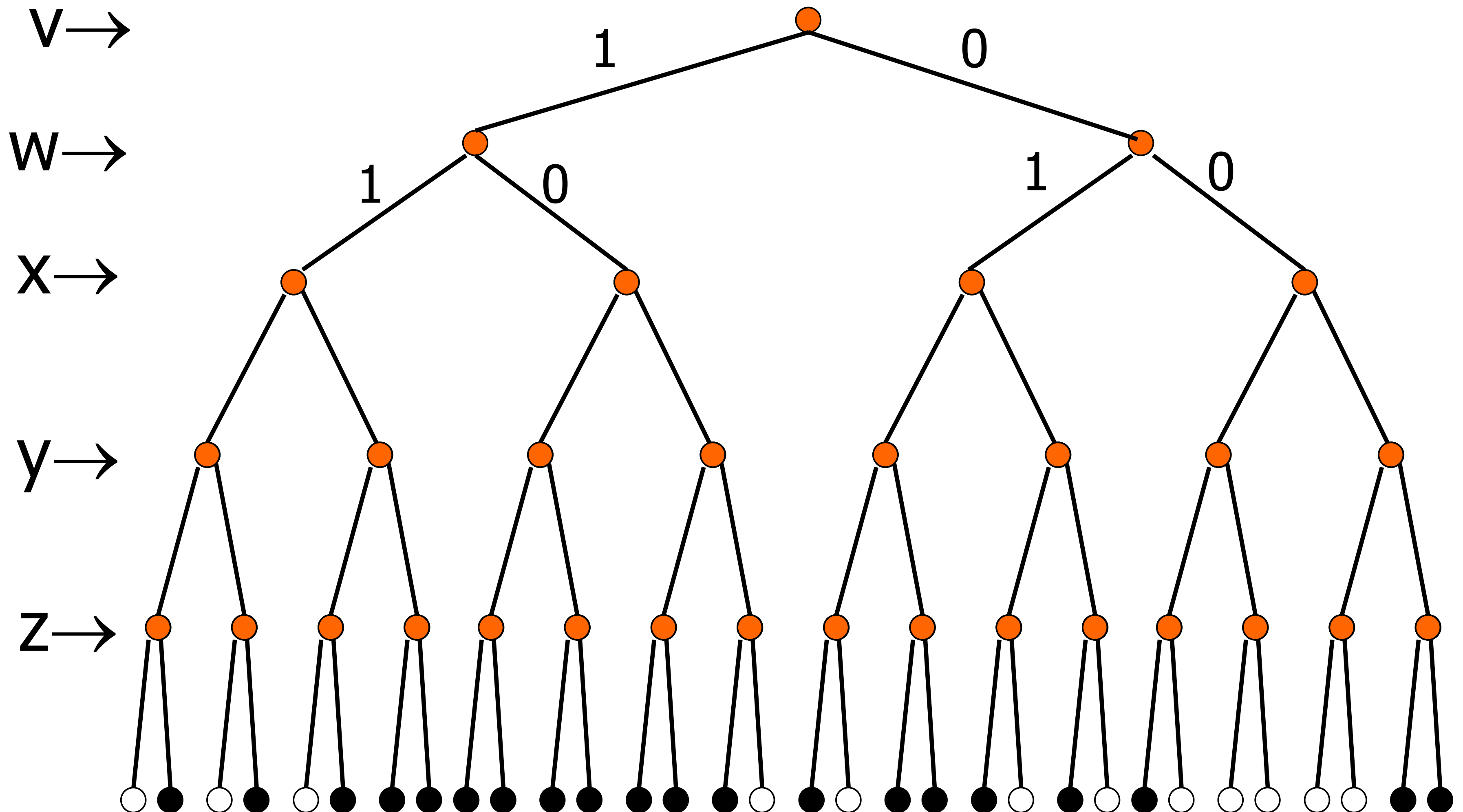
2-bit addition under the microscope

- $\sigma(y) = \langle 1,1,0 \rangle$ so 3 is untainted
- $\sigma(z) = \langle 1,0,0 \rangle$ so 1 is untainted
- Calculate the carry bits:
  - $c_1 = 0$
  - $c_2 = (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 0) = 1$
  - $c_3 = (1 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 1) = 1$
- Calculate the sum bits:
  - $b_1 = 1 \oplus 1 \oplus 0 = 0$
  - $b_2 = 1 \oplus 0 \oplus 1 = 0$
- Compute the taint bit:
  - $b_3 = 0 \vee 0 \vee 1 = 1$

Formalising a hack

- Rules for ==, !, || and && can be defined with
  - rules for <=, | and &
- Let $C \to^k C'$ iff $C'=\{c' \mid c \in C \text{ and } c \to^k c'\}$
- Then $hack^k(s) = \{\sigma \mid \ldots\}$
  - $\{\langle s, \sigma\rangle\} \to^k \{\langle (x = malloc(y); s'), \sigma'\rangle\}$ and
  - $\sigma'(y)^{w+1} = true$
- With $\sigma \in hack^k(s)$, s will *surely* encounter an overflow fed malloc at step k
- Note that $\sigma \in hack^{42}(s)$ is simpler to check and, thus exploit, than $\sigma' \in hack^{123}(s)$

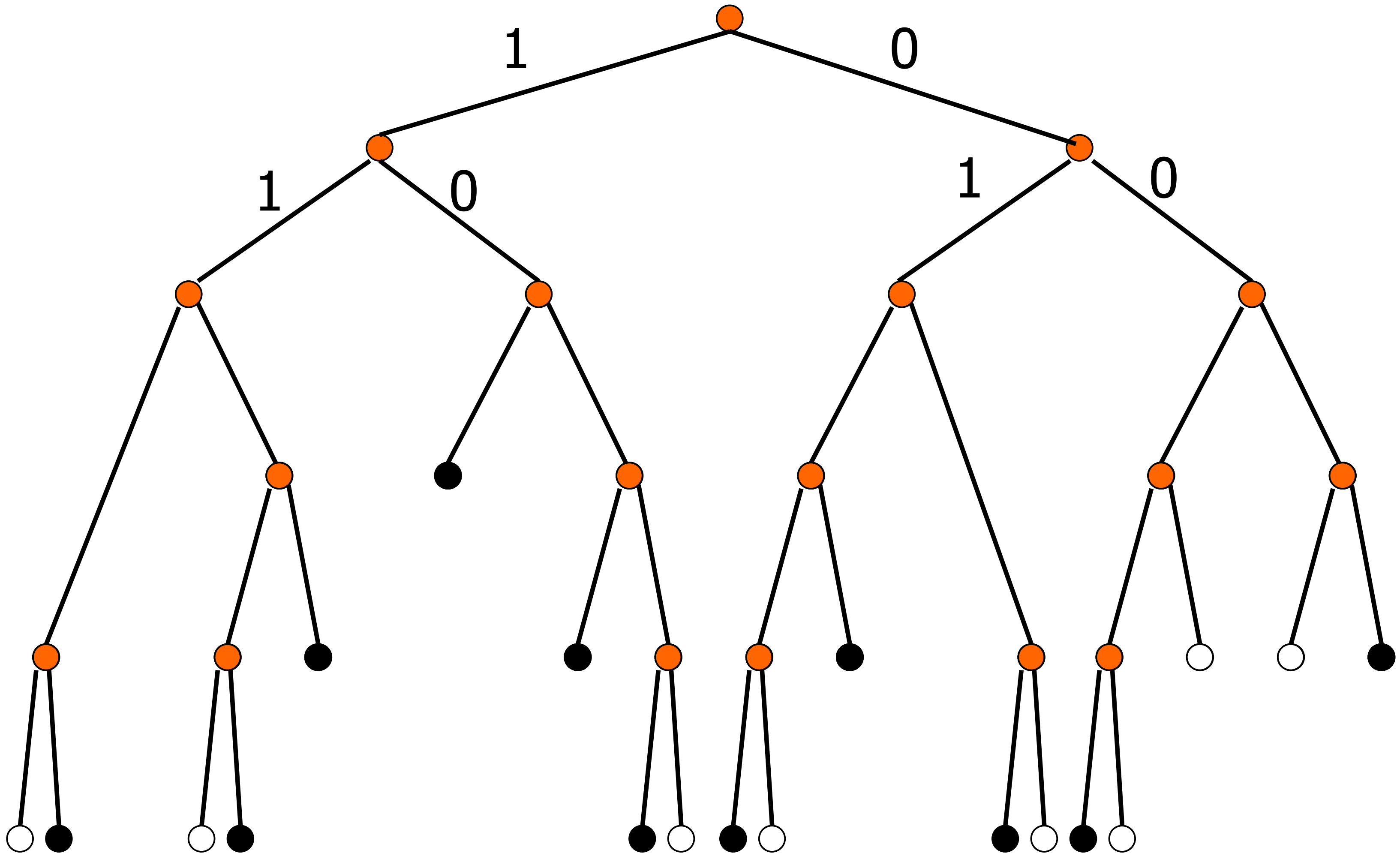
OBDD: an API on bit-operations



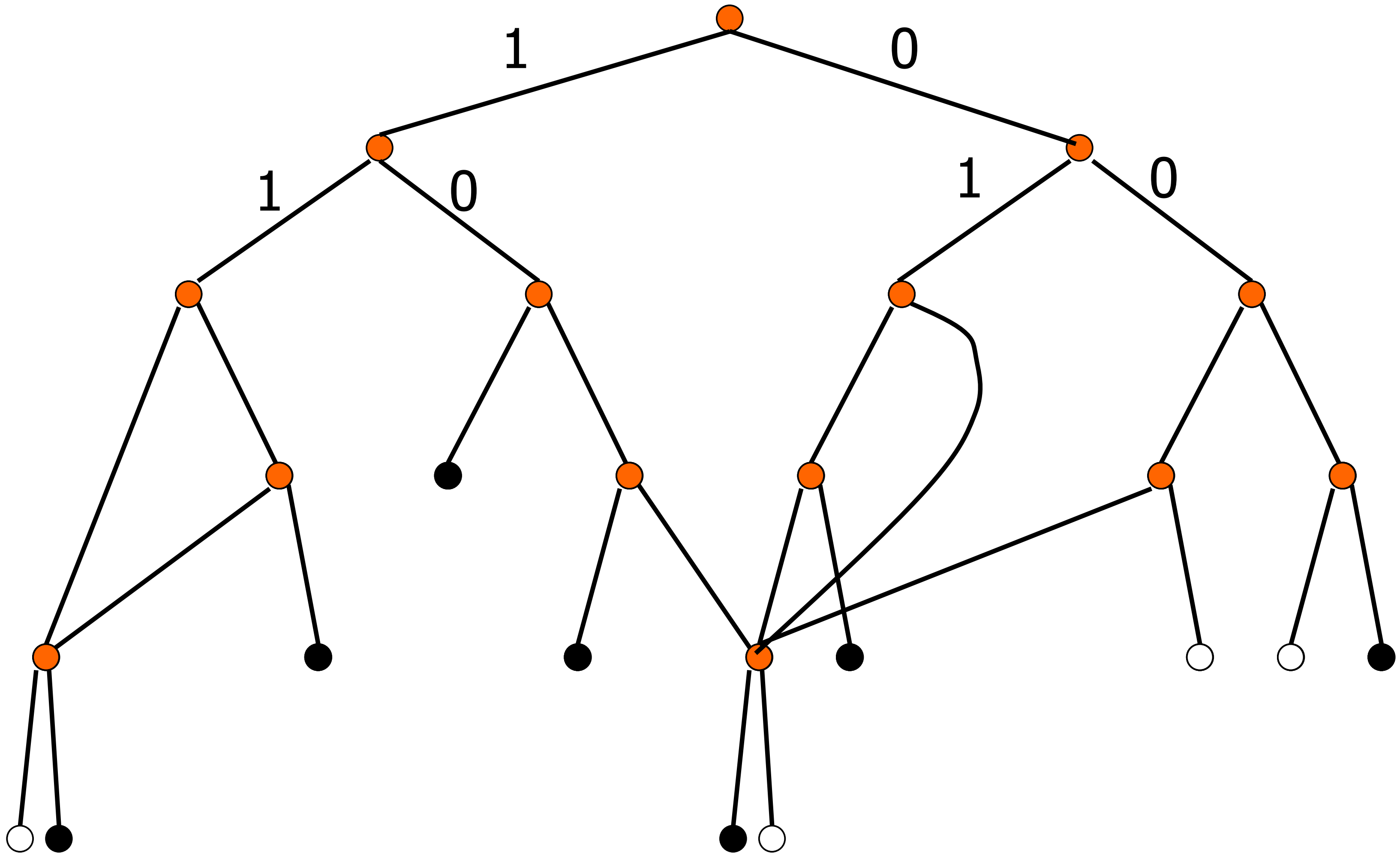
ROBDD (reduction 1)

1 0
1 0 1 0

ROBDD (reduction 2)



ROBDD (reduction 3)



Encoding taint semantics in ROBDDs [Hawkins, JAIR, 2005]

- Use function f to encode behaviour of s by:
  - if $\langle s, \sigma \rangle \rightarrow \langle skip, \sigma' \rangle$ then
  - $f = \alpha(\sigma) \wedge \alpha'(\sigma')$ where
  - $\alpha(\sigma) = \wedge_{x \in var(s)} \wedge_{i \in [1,w+1]} x_i \leftrightarrow \sigma(x)^i$
  - $\alpha'(\sigma') = \wedge_{x \in var(s)} \wedge_{i \in [1,w+1]} x_i' \leftrightarrow \sigma'(x)^i$
- If s = (x = y & z) then $[[s]] = \alpha(\sigma) \wedge \alpha'(\sigma') =$
  - $(\wedge_{i \in [1,w]} x_i' \leftrightarrow (y_i \wedge z_i)) \wedge$
  - $(x_{w+1}' \leftrightarrow (y_{w+1} \vee z_{w+1})) \wedge$
  - $(\wedge_{i \in [1,w+1]} y_i' \leftrightarrow y_i) \wedge (\wedge_{i \in [1,w+1]} z_i' \leftrightarrow z_i)$

Ensuring a taint

- Consider x = y + z; w = malloc(x)
- Interested in values of x, y and z which induce an overflow:
  - $f \wedge [[x = y + z]] \models x'_{33}$ where f is defined over $x_1, \ldots z_{33}$
  - $f \models ([[x = y + z]] \rightarrow x'_{33})$ where f is defined over $x_1, \ldots z_{33}$
  - $f = \forall_{x'_1} \ldots \forall_{z'_{33}} ([[x = y + z]] \rightarrow x'_{33})$ where $\forall_y(g) = g[0/y] \wedge g[1/y]$
- Mix of constraint solving, deduction with …
  - If $[[x = y + z]] \models g$ then use g instead of [[…]]

Need for approximation

- Prototype implementation:
  - ROBDD explosion on <<
- Tactics for tractability:
  - Variable reordering [Rudell, ICCAD, 1993]
  - Early projection [Vardi, CP, 2001]
    - $||\exists_x(f)|| < ||f||$ where $||.||$ counts nodes
  - ROBDD approximation [Ravi, DAC, 1998]
    - $||f|| > ||g||$ where $f \models g$
  - ROBDD approximation [Kettle, TACAS, 2006]
    - $||f|| \approx 2||g||$ where $f \models g$

The End