Dualising effect systems to understand resources and context dependence

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Context in programming

and its (static) analysis

- **Free variables (and their types)**

  Simply-typed λ-calculus $\Gamma \vdash e : \tau$

  $\{x_1 : \tau_1, \ldots, x_n : \tau_n\}$

- **Free variables (types and usage)**

  Linear types $\Gamma \vdash e : \sigma \multimap \tau$

  Bounded linear types $\Gamma \vdash e : !_{n}\sigma \rightarrow \tau$

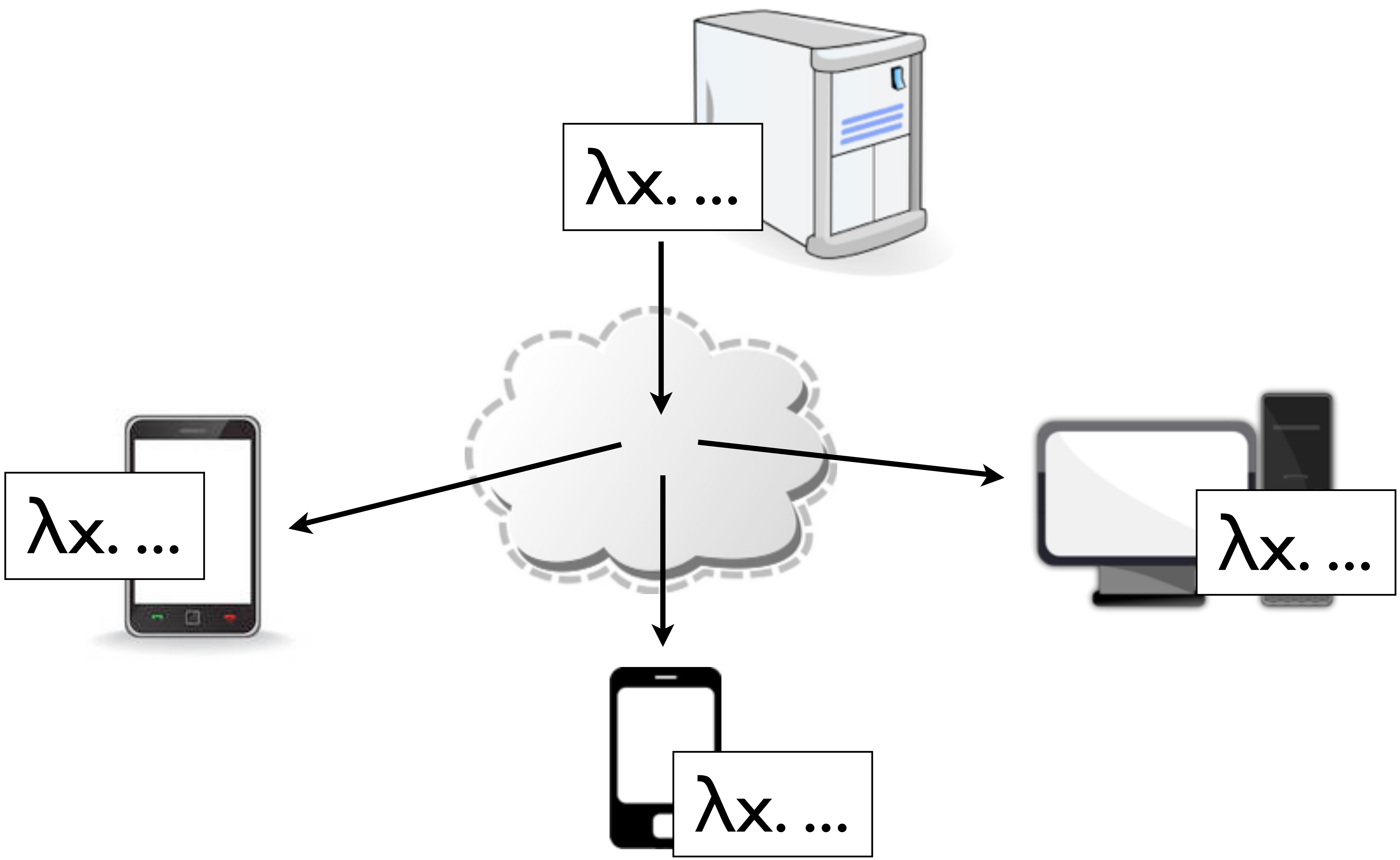
- **Compilation context (e.g., overloading)**

  Type class constraints $(+) :: \mathtt{Num\ a} \Rightarrow \mathtt{a} \rightarrow \mathtt{a} \rightarrow \mathtt{a}$

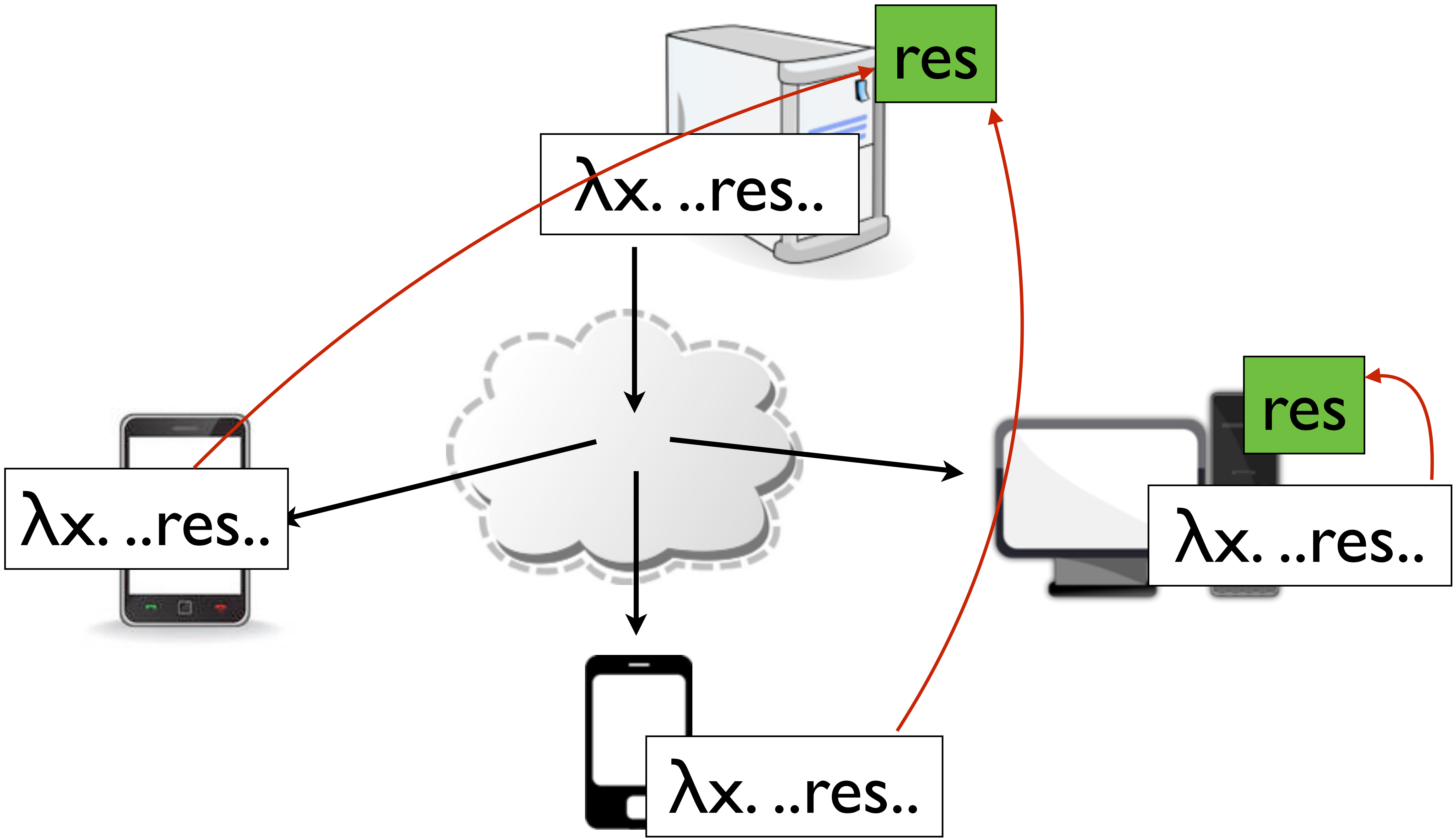
- **Run-time context (e.g., resources)**

  Input-output requirements, sensors, databases…

Context: distributed programming



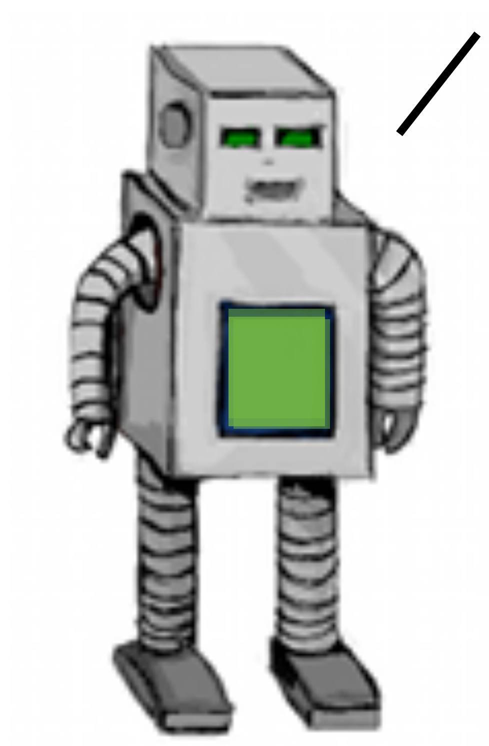
Context: distributed programming



The *coeffect calculus*

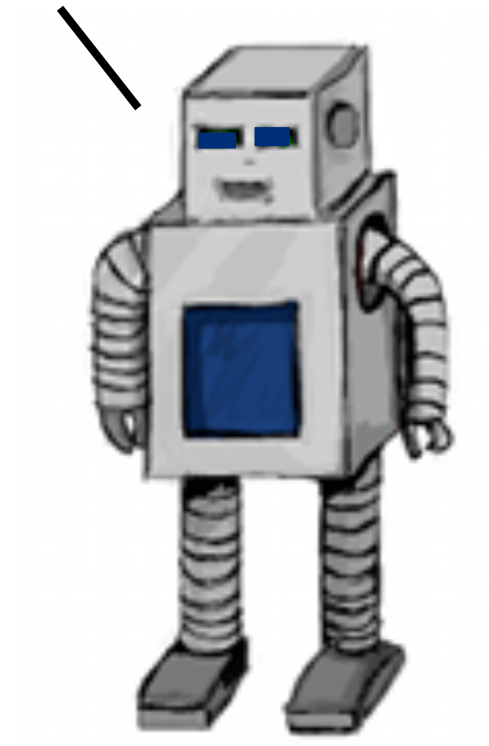
- Augmented simply-typed λ-calculus
- for contextual features:
  - generalised analysis via a type and *coeffect system*
  - generalised denotational semantics via *indexed comonads*
- Coeffect : d*ualises* effects (+ monads)

“how”



$expr$

“what”



coeffects

types

Effect systems

$$\Gamma \vdash e : \tau, \mathsf{F}$$

$$\text{abs}\ \frac{\Gamma, x : \sigma \vdash e : \tau, \mathsf{F}}{\Gamma \vdash \lambda x\,.\, e : \sigma \xrightarrow{\mathsf{F}} \tau, \varnothing} \qquad \text{var}\ \frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma, \varnothing}$$

$$\text{app}\ \frac{\Gamma \vdash e_1 : \sigma \xrightarrow{\mathsf{F}} \tau, \mathsf{G} \qquad \Gamma \vdash e_2 : \sigma, \mathsf{H}}{\Gamma \vdash e_1\, e_2 : \tau, \mathsf{F} \sqcup \mathsf{G} \sqcup \mathsf{H}}$$

(semi)lattice $(\mathsf{F}, \sqcup, \varnothing)$

$$\text{write}\ \frac{\Gamma \vdash e : \tau, \mathsf{F} \qquad (x : ref\ \tau) \in \Gamma}{\Gamma \vdash x := e : (), \mathsf{F} \cup \{\mathrm{W}(x)\}} \qquad \text{read}\ \frac{(x : ref\ \tau) \in \Gamma}{\Gamma \vdash\ !x : \tau, \{\mathrm{R}(x)\}}$$

Monads

combine effects

$$\text{comp}\ \frac{f : \mathrm{X} \to \mathbf{M}\,\mathrm{Y} \qquad g : \mathrm{Y} \to \mathbf{M}\,\mathrm{Z}}{(f;g) : \mathrm{X} \to \mathbf{M}\,\mathrm{Z}}$$

$$\text{id}\ \frac{}{id_{\mathrm{A}} : \mathrm{A} \to \mathbf{M}\,\mathrm{A}}$$

trivial effect

$$[\ \Gamma \vdash e : \tau\ ] : [\Gamma] \to \mathbf{M}[\tau]$$

$$[\sigma \to \tau] = [\sigma] \to \mathbf{M}[\tau]$$

For ease here: $\Gamma \vdash e : \mathbf{M}\ \tau$

Monads

$$\Gamma \vdash e : \mathbf{M}\tau$$

$$\text{abs}\ \frac{\Gamma, x : \sigma \vdash e : \mathbf{M}\tau}{\Gamma \vdash \lambda x\,.\, e : \mathbf{M}(\sigma \to \mathbf{M}\tau)}$$

$$\text{var}\ \frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \mathbf{M}\sigma}$$

$$\text{app}\ \frac{\Gamma \vdash e_1 : \mathbf{M}\,(\sigma \to \mathbf{M}\tau) \qquad \Gamma \vdash e_2 : \mathbf{M}\sigma}{\Gamma \vdash e_1\, e_2 : \mathbf{M}\tau}$$

Parametric effect monads

$$\text{comp}\ \frac{f : X \to \mathbf{M}_{F} Y \qquad g : Y \to \mathbf{M}_{G} Z}{(f;g)\ : X \to \mathbf{M}_{(F \sqcup G)} Z}$$

$$\text{id}\ \frac{}{id_A\ : A \to \mathbf{M}_{\varnothing} A}$$

(semi)lattice/monoid $(F, \sqcup, \varnothing)$

$$[\ \Gamma \vdash\ e\ :\ \tau,\ F\ ]\ :\ [\Gamma] \to \mathbf{M}_F[\tau]$$

$$[\sigma \xrightarrow{F} \tau] = [\sigma] \to \mathbf{M}_F[\tau]$$

[see Katsumata, POPL 2014]

Monads + effects

$$\Gamma \vdash e : \mathbf{M}_{F}\tau$$

$$\text{abs}\ \frac{\Gamma, x : \sigma \vdash e : \mathbf{M}_{F}\tau}{\Gamma \vdash \lambda x\,.\,e : \mathbf{M}_{\emptyset}(\sigma \rightarrow \mathbf{M}_{F}\tau)}$$

$$\text{var}\ \frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \mathbf{M}_{\emptyset}\sigma}$$

lambda delays effects

$$\text{app}\ \frac{\Gamma \vdash e_1 : \mathbf{M}_{F}(\sigma \rightarrow \mathbf{M}_{G}\tau) \qquad \Gamma \vdash e_2 : \mathbf{M}_{H}\sigma}{\Gamma \vdash e_1\, e_2 : \mathbf{M}_{(F \sqcup G \sqcup H)}\tau}$$

[see Katsumata, POPL 2014]

Dualising

$$A \leftarrow B \quad \Big\| \quad A \rightarrow B$$

Productive/output effects $[\![\Gamma \vdash e : \tau, \mathsf{F}]\!] : [\![\Gamma]\!] \rightarrow \mathbf{M}_{\mathsf{F}}[\![\tau]\!]$

structure on the right

Consuming/input effects $[\![\Gamma \mathrel{?} \mathsf{R} \vdash e : \tau]\!] : \mathbf{C}_{\mathsf{R}}[\![\Gamma]\!] \rightarrow [\![\tau]\!]$

Coeffects

structure on the left

But… λ-calculus is asymmetric: many-to-one

Design question: structure over all Γ or per variable?

answer : yes! either

Coeffects: two varieties (shapes)

$$\Gamma \; ? \; \mathsf{R} \vdash e : \tau$$

per variable

shape = $|\Gamma|$

$\mathsf{R} = \langle r_1, \ldots, r_n \rangle$

$\textit{where } \Gamma = \{x_1 : \tau_1, \ldots, x_n : \tau_n\}$

$$\mathsf{C}_{r1}[\tau_1] \times \ldots \times \mathsf{C}_{rn}[\tau_n] \rightarrow [\tau]$$

per context

shape = 1

$\mathsf{R} = r_1$

$$\mathsf{C}_{r1}[\Gamma] \rightarrow [\tau]$$

Example: reuse bounds bounded linear logic

- Per variable
- Track number of uses of a variable
- Coeffects annotations are vectors of natural numbers

Example: reuse bounds bounded linear logic

abs
$$\frac{\Gamma, x : \sigma \,?\, R \times \langle s \rangle \vdash e : \tau}{\Gamma \,?\, R \vdash \lambda x.e : \sigma \xrightarrow{s} \tau}$$

var
$$\frac{}{x : \sigma \,?\, \langle 1 \rangle \vdash x : \sigma}$$

app
$$\frac{\Gamma_1 \,?\, R \vdash e_1 : \sigma \xrightarrow{t} \tau \qquad \Gamma_2 \,?\, S \vdash e_2 : \sigma}{\Gamma_1, \Gamma_2 \,?\, R \times (t * S) \vdash e_1\, e_2 : \tau}$$

contr
$$\frac{\Gamma_1, x : \sigma,\; y : \sigma,\; \Gamma_2 \,?\, R \times \langle a,b \rangle \times S \vdash e : \tau}{\Gamma_1,\; z : \sigma,\; \Gamma_2 \,?\, R \times \langle a+b \rangle \times S \vdash e : \tau}$$

weak
$$\frac{\Gamma \,?\, R \vdash e : \tau}{\Gamma, x : \tau \,?\, R \times \langle 0 \rangle \vdash e : \tau}$$

sub
$$\frac{\Gamma \,?\, R \vdash e : \tau \qquad R \leq R'}{\Gamma \,?\, R' \vdash e : \tau}$$

Example: reuse bounds

$$\text{const}\ \frac{}{\varnothing \mathbin{?} \langle\rangle \vdash c : t}$$

$$\text{exchg}\ \frac{\Gamma_1, x : \sigma_1,\ y : \sigma_2, \Gamma_2 \mathbin{?} \mathsf{R} \times \langle \mathsf{a,b} \rangle \times \mathsf{S} \vdash e : \tau}{\Gamma_1,\ y : \sigma_2, x : \sigma_1,\ \Gamma_2 \mathbin{?} \mathsf{R} \times \langle \mathsf{b,a} \rangle \times \mathsf{S} \vdash e : \tau}$$

Example: reuse bounds

$$(\lambda v.x+v+v)\ (x+y)$$

$$\dfrac{(\text{app})\dfrac{(\text{abs})\dfrac{x:\mathbb{Z}, v:\mathbb{Z}?\langle 1,2\rangle \vdash x+v+v:\mathbb{Z}}{x:\mathbb{Z}?\langle 1\rangle \vdash (\lambda v.x+v+v):\mathbb{Z}\xrightarrow{2}\mathbb{Z}} \qquad \dfrac{\vdots}{x':\mathbb{Z}, y:\mathbb{Z}?\langle 1,1\rangle \vdash x'+y:\mathbb{Z}}}{(\equiv)\dfrac{x:\mathbb{Z}, x':\mathbb{Z}, y:\mathbb{Z}?\langle 1\rangle \times (2*\langle 1,1\rangle) \vdash (\lambda v.x+v+v)\ (x'+y):\mathbb{Z}}{(\text{contr})\dfrac{x:\mathbb{Z}, x':\mathbb{Z}, y:\mathbb{Z}?\langle 1,2,2\rangle \vdash (\lambda v.x+v+v)\ (x'+y):\mathbb{Z}}{x:\mathbb{Z}, y:\mathbb{Z}?\langle 3,2\rangle \vdash (\lambda v.x+v+v)\ (x+y):\mathbb{Z}}}}{}$$

$$(\lambda v.x+v+v)\ (x+y) \ \rightsquigarrow_\beta \ x+(x+y)+(x+y)$$

$$x:\mathbb{Z}, y:\mathbb{Z}?\langle 3,2\rangle \vdash x+(x+y)+(x+y):\mathbb{Z}$$

Example: resources

- Per context

- Tracks resource requirements, e.g.

$$\varnothing ? \{\mathsf{gps}\} \vdash \textbf{access}\ \mathsf{gps}\ : \mathrm{Coord}$$

- "impure" lambda

$$\varnothing ? \{\mathsf{gps}\} \vdash (\mathsf{fn}\ x\ .\ \textbf{access}\ \mathsf{gps} + x)\ : \mathrm{Int} \xrightarrow{\{\}} \mathrm{Coord}$$

gps at declaration site

$$\varnothing ? \{\} \vdash (\mathsf{fn}\ x\ .\ \textbf{access}\ \mathsf{gps} + x)\ : \mathrm{Int} \xrightarrow{\{\mathsf{gps}\}} \mathrm{Coord}$$

gps at call site

Example: resources

$$\text{abs}\ \frac{\Gamma, x : \sigma \mathbin{?} \mathsf{R} \cup \mathsf{S} \vdash e : \tau}{\Gamma \mathbin{?} \mathsf{R} \vdash \lambda x.e : \sigma \xrightarrow{\mathsf{S}} \tau}$$

$$\text{var}\ \frac{}{x : \sigma \mathbin{?} \varnothing \vdash x : \sigma}$$

$$\text{app}\ \frac{\Gamma_1 \mathbin{?} \mathsf{R} \vdash e_1 : \sigma \xrightarrow{\mathsf{T}} \tau \qquad \Gamma_2 \mathbin{?} \mathsf{S} \vdash e_2 : \sigma}{\Gamma_1, \Gamma_2 \mathbin{?} \mathsf{R} \cup \mathsf{S} \cup \mathsf{T} \vdash e_1\ e_2 : \tau}$$

$$\text{contr}\ \frac{\Gamma_1, x : \tau, y : \tau, \Gamma_2 \mathbin{?} \mathsf{R} \vdash e : \tau}{\Gamma_1, z : \tau, \Gamma_2 \mathbin{?} \mathsf{R} \vdash e : \tau}$$

$$\text{access}\ \frac{\mathbf{R} : \tau}{\Gamma, x : \tau \mathbin{?} \{\mathsf{R}\} \vdash \mathbf{access\ R} : \tau}$$

$$\text{sub}\ \frac{\Gamma \mathbin{?} \mathsf{R} \vdash e : \tau \qquad \mathsf{R} \subseteq \mathsf{R'}}{\Gamma \mathbin{?} \mathsf{R'} \vdash e : \tau}$$

Example: resources

$$\text{abs}\ \frac{\Gamma, x : \sigma \,?\, \mathsf{R} \cup \mathsf{S} \vdash e : \tau}{\Gamma \,?\, \mathsf{R} \vdash \lambda x.e : \sigma \xrightarrow{\mathsf{S}} \tau}$$

introduces non-determinism

$$\varnothing ? \{\mathsf{gps}\} \vdash (\mathsf{fn}\ x\ .\ \textbf{access}\ \mathsf{gps} + x)\ :\ \mathrm{Int} \xrightarrow{\{\}} \mathrm{Coord}$$ declaration site

$$\varnothing ? \{\} \vdash (\mathsf{fn}\ x\ .\ \textbf{access}\ \mathsf{gps} + x)\ :\ \mathrm{Int} \xrightarrow{\{\mathsf{gps}\}} \mathrm{Coord}$$ call site

$$\text{signature}\ \frac{\Gamma \,?\, \mathsf{R} \vdash e : \tau}{\Gamma \,?\, \mathsf{R} \vdash (e : \tau) : \tau}$$

Example: resources

$$\text{abs}\ \frac{\Gamma,\ x : \sigma\ ?\ \mathsf{R} \cup \mathsf{S} \vdash e : \tau}{\Gamma\ ?\ \mathsf{R} \vdash \lambda x.e : \sigma \xrightarrow{\mathsf{S}} \tau}$$

$$\text{var}\ \frac{}{x : \sigma\ ?\ \varnothing \vdash x : \sigma}$$

$$\text{app}\ \frac{\Gamma_1\ ?\ \mathsf{R} \vdash e_1 : \sigma \xrightarrow{\mathsf{T}} \tau \qquad \Gamma_2\ ?\ \mathsf{S} \vdash e_2 : \sigma}{\Gamma_1, \Gamma_2\ ?\ \mathsf{R} \cup \mathsf{S} \cup \mathsf{T} \vdash e_1\ e_2 : \tau}$$

$$\text{contr}\ \frac{\Gamma_1,\ x : \tau,\ y : \tau,\ \Gamma_2\ ?\ \mathsf{R} \vdash e : \tau}{\Gamma_1,\ z : \tau,\ \Gamma_2\ ?\ \mathsf{R} \vdash e : \tau}$$

$$\text{weak}\ \frac{\Gamma\ ?\ \mathsf{R} \vdash e : \tau}{\Gamma,\ x : \tau\ ?\ \mathsf{R} \vdash e : \tau}$$

$$\text{sub}\ \frac{\Gamma\ ?\ \mathsf{R} \vdash e : \tau \qquad \mathsf{R} \subseteq \mathsf{R'}}{\Gamma\ ?\ \mathsf{R'} \vdash e : \tau}$$

Example: reuse bounds

abs
$$\frac{\Gamma,\ x : \sigma\ ?\ \mathsf{R} \times \langle \mathsf{s} \rangle \vdash e : \tau}{\Gamma\ ?\ \mathsf{R} \vdash \lambda x.e : \sigma \xrightarrow{\mathsf{s}} \tau}$$

var
$$\frac{}{x : \sigma\ ?\ \langle 1 \rangle \vdash x : \sigma}$$

app
$$\frac{\Gamma_1\ ?\ \mathsf{R} \vdash e_1 : \sigma \xrightarrow{\mathsf{t}} \tau \qquad \Gamma_2\ ?\ \mathsf{S} \vdash e_2 : \sigma}{\Gamma_1, \Gamma_2\ ?\ \mathsf{R} \times (\mathsf{t} * \mathsf{S}) \vdash e_1\ e_2 : \tau}$$

contr
$$\frac{\Gamma_1,\ x : \sigma,\ y : \sigma,\ \Gamma_2\ ?\ \mathsf{R} \times \langle \mathsf{a,b} \rangle \times \mathsf{S} \vdash e : \tau}{\Gamma_1,\ z : \sigma,\ \Gamma_2\ ?\ \mathsf{R} \times \langle \mathsf{a+b} \rangle \times \mathsf{S} \vdash e : \tau}$$

weak
$$\frac{\Gamma\ ?\ \mathsf{R} \vdash e : \tau}{\Gamma,\ x : \tau\ ?\ \mathsf{R} \times \langle 0 \rangle \vdash e : \tau}$$

sub
$$\frac{\Gamma\ ?\ \mathsf{R} \vdash e : \tau \qquad \mathsf{R} \leq \mathsf{R'}}{\Gamma\ ?\ \mathsf{R'} \vdash e : \tau}$$

combine per-variable and per-context

Coeffect calculus

$$\text{abs}\ \frac{\Gamma, x : \sigma \ ?\ R \ltimes \langle s \rangle \vdash e : \tau}{\Gamma \ ?\ R \vdash \lambda x.e : \sigma \xrightarrow{s} \tau}$$

$$\text{var}\ \frac{}{x : \sigma \ ?\ \langle use \rangle \vdash x : \sigma}$$

$$\text{app}\ \frac{\Gamma_1 \ ?\ R \vdash e_1 : \sigma \xrightarrow{t} \tau \qquad \Gamma_2 \ ?\ S \vdash e_2 : \sigma}{\Gamma_1, \Gamma_2 \ ?\ R \rtimes (t \circledast S) \vdash e_1\, e_2 : \tau}$$

$$\text{contr}\ \frac{\Gamma_1, x : \sigma, y : \sigma, \Gamma_2 \ ?\ R \ltimes \langle a \rangle \ltimes \langle b \rangle \ltimes S \vdash e : \tau}{\Gamma_1, z : \sigma, \Gamma_2 \ ?\ R \rtimes \langle a \oplus b \rangle \rtimes S \vdash e : \tau}$$

$$\text{weak}\ \frac{\Gamma \ ?\ R \vdash e : \tau}{\Gamma, x : \tau \ ?\ R \rtimes \langle ign \rangle \vdash e : \tau}$$

$$\text{sub}\ \frac{\Gamma \ ?\ R \vdash e : \tau \qquad R \leq R'}{\Gamma \ ?\ R' \vdash e : \tau}$$

Coeffect calculus

- Coeffect scalar $(C, \circledast, \oplus, use, ign, \leq)$
  - $\circledast$ - sequential compose
  - $\oplus$ - share (contraction)
  - use - variables
  - ign - null
  - $\leq$ - subcoeffecting
- Coeffect shapes $(S, [-], \diamond, \underline{0}, \underline{1})$
  - $[-]$ - context to shape
  - $\diamond$ - compose shapes
  - $\underline{0}$ - empty
  - $\underline{1}$ - singleton
- Coeffect algebra $(\ltimes, \rtimes, \perp)$
  - $\ltimes : C^n \times C^m \rightarrow C^{n \diamond m}$ - merge coeffects in premise
  - $\rtimes : C^n \times C^m \rightarrow C^{n \diamond m}$ - merge coeffects in conclusion
  - $\perp : C^{\underline{0}}$ - empty context
  - $\langle\ \rangle : C \rightarrow C^{\underline{1}}$ - lift to shaped
  - $\circledast : C \times C^m \rightarrow C^m$ - scalar-shaped compose

combine per-variable and per-context

Coeffect calculus

$$\text{abs}\ \frac{\Gamma, x : \sigma \ ?\ R \ltimes \langle s \rangle \vdash e : \tau}{\Gamma \ ?\ R \vdash \lambda x.e : \sigma \xrightarrow{s} \tau}$$

$$\text{var}\ \frac{}{x : \sigma \ ?\ \langle use \rangle \vdash x : \sigma}$$

$$\text{app}\ \frac{\Gamma_1 \ ?\ R \vdash e_1 : \sigma \xrightarrow{t} \tau \qquad \Gamma_2 \ ?\ S \vdash e_2 : \sigma}{\Gamma_1, \Gamma_2 \ ?\ R \rtimes (t \circledast S) \vdash e_1\, e_2 : \tau}$$

$$\text{contr}\ \frac{\Gamma_1, x : \sigma, y : \sigma, \Gamma_2 \ ?\ R \ltimes \langle a \rangle \ltimes \langle b \rangle \ltimes S \vdash e : \tau}{\Gamma_1, z : \sigma, \Gamma_2 \ ?\ R \rtimes \langle a \oplus b \rangle \rtimes S \vdash e : \tau}$$

$$\text{weak}\ \frac{\Gamma \ ?\ R \vdash e : \tau}{\Gamma, x : \tau \ ?\ R \rtimes \langle ign \rangle \vdash e : \tau}$$

$$\text{sub}\ \frac{\Gamma \ ?\ R \vdash e : \tau \qquad R \leq R'}{\Gamma \ ?\ R' \vdash e : \tau}$$

Comparing coeffects & effects

$$\frac{\Gamma,\ x : \sigma \mathrel{?} \mathsf{R} \ltimes \langle \mathsf{s} \rangle \vdash e : \tau}{\Gamma \mathrel{?} \mathsf{R} \vdash \lambda x.e : \sigma \xrightarrow{\mathsf{s}} \tau}$$

$$\frac{\Gamma,\ x : \sigma \vdash e : \tau,\ \mathsf{F}}{\Gamma \vdash \lambda x\ .\ e : \sigma \xrightarrow{\mathsf{F}} \tau,\ \emptyset}$$

$$\frac{\Gamma,\ x : \sigma \mathrel{?} \mathsf{R} \times \langle \mathsf{s} \rangle \vdash e : \tau}{\Gamma \mathrel{?} \mathsf{R} \vdash \lambda x.e : \sigma \xrightarrow{\mathsf{s}} \tau}$$

reuse bounds

$$\frac{\Gamma,\ x : \sigma \mathrel{?} \mathsf{R} \cup \mathsf{S} \vdash e : \tau}{\Gamma \mathrel{?} \mathsf{R} \vdash \lambda x.e : \sigma \xrightarrow{\mathsf{S}} \tau}$$

resources

Semantics…

- Effect calculus
  - Indexed strong monad — sequential composition + plumbing
- Coeffect calculus
  - Indexed comonad — sequential composition
  - Indexed structural comonad — + plumbing
  - Indexed lax & colax monoidal structure — context merge/split

recall

Parametric effect monads

$$\text{comp}\ \frac{f : X \to \mathbf{M}_{F} Y \qquad g : Y \to \mathbf{M}_{G} Z}{(f;g) : X \to \mathbf{M}_{(F \sqcup G)} Z}$$

$$\text{id}\ \frac{}{id_A : A \to \mathbf{M}_{\emptyset} A}$$

(semi)lattice/monoid $(F, \sqcup, \emptyset)$

$$[\ \Gamma \vdash e : \tau, F\ ] : [\Gamma] \to \mathbf{M}_{F}[\tau]$$

$$[\sigma \xrightarrow{F} \tau] = [\sigma] \to \mathbf{M}_{F}[\tau]$$

[see Katsumata, POPL 2014]

Indexed comonad (parametric effect comonad)

$$\text{comp}\ \frac{f : \mathsf{C}_{r} X \to Y \qquad\qquad g : \mathsf{C}_{s} Y \to Z}{(f;g) : \mathsf{C}_{r \circledast s} X \to Z}$$

$$\text{id}\ \frac{}{id_A : \mathsf{C}_{use} A \to A}$$

(semi)lattice/monoid $(R, \circledast, use)$

$[\ \Gamma\ ?\ r \vdash\ e : \tau\ ] : \mathsf{C}'_{r}[\Gamma] \to [\tau]$

$[\sigma \xrightarrow{r} \tau] = \mathsf{C}_{r}[\sigma] \to [\tau]$

[see Petricek, Orchard, Mycroft, ICALP 2013, ICFP 2014]

Indexed structural comonad

$$\text{comp}\ \frac{f : \mathsf{C}^{n}_{R}(X_1,..X_n) \to Y \quad g : \mathsf{C}^{1}_{s} Y \to Z}{(f;g) : \mathsf{C}^{n}_{s \circledast R}(X_1,..Xn) \to Z}$$

$$\text{id-left}\ \frac{}{id_A : \mathsf{C}^{0}_{\perp} A \to A} \qquad \text{id-right}\ \frac{}{id_A : \mathsf{C}^{I}_{use} A \to A}$$

monoid left-action $(R^n, \circledast)$

$$[\ \Gamma \ ?\ R \vdash e : \tau\ ] : \mathsf{C}^{|\Gamma|}_{R}[\Gamma] \to [\tau]$$

[see Petricek, Orchard, Mycroft, ICFP 2014]

Breakout: comparing semantics

$$\frac{\Gamma,\, x : \sigma \; ? \; \mathsf{R} \ltimes \langle \mathsf{s} \rangle \vdash e : \boldsymbol{\tau}}{\Gamma \; ? \; \mathsf{R} \vdash \lambda x.e : \sigma \xrightarrow{\mathsf{s}} \boldsymbol{\tau}}$$

$$\frac{\Gamma,\, x : \sigma \vdash e : \boldsymbol{\tau},\ \mathsf{F}}{\Gamma \vdash \lambda x \;.\; e : \sigma \xrightarrow{\mathsf{F}} \boldsymbol{\tau},\ \varnothing}$$

Merging and splitting

$$\text{merge}\ \frac{f : \mathbf{C}^{n}_{R}(X_1,..,X_n) \qquad g : \mathbf{C}^{m}_{S}(Y_1,..,Y_n)}{(\mathrm{merge}\ f\ g)\ : \mathbf{C}^{n \diamond m}_{R \ltimes S}(X_1,..,X_n, Y_1,..,Y_n)}$$

$$\text{split}\ \frac{f : \mathbf{C}^{n \diamond m}_{R \rtimes S}(X_1,..,X_n, Y_1,..,Y_n)}{(\mathrm{split}\ f)\ :\ \mathbf{C}^{n}_{R}(X_1,..,X_n) \times \mathbf{C}^{m}_{S}(Y_1,..,Y_n)}$$

What's next

- Bieffects $\Gamma \,?\, R \vdash e : \tau, F$
- Extend to other calculi (pi-calculus?)
- Extend to program logics (partial operations)
- New examples: security? information flow?
- Reused semantic derivation techniques for other types

Conclusions

- Dualising effects is subtle due to asymmetry
- Coeffects require more algebraic structure
- Lots of interesting examples being discovered

- Lots of new work:
  - Brunel, Gaboardi, Mazza, Zdancewic "*A core quantitative coeffect calculus*" (ESOP 2014)
  - Ghica, Smith "*Bounded linear types in a resource semiring*" (ESOP 2014)
  - Petricek, Orchard, Mycroft "*Coeffects: a calculus of context-dependent computation*" (ICFP 2014)
  - Orchard, Petricek "*Embedding effect systems in Haskell*" (Haskell, 2014)

Thanks!

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Tomas working on his
upcoming thesis “Context-aware programming languages”