Effect systems revisited – control-flow algebra and semantics (slides)

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for our example we get $\{G3, F3, A3, B3, C4, D4, E4, G4\}$

...in POPL'94, richer effect systems for concurrency

Higher-Order Concurrent Programs with Finite Communication Topology

(Extended Abstract)

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Abstract

Concurrent ML (CML) is an extension of the functional language Standard ML (SML) with primitives for the dynamic creation of processes and channels and for the communication of values over channels. Because of the powerful abstraction mechanisms the communication topology of a given program may be very complex and therefore an efficient implementation may be facilitated by knowledge of the topology.

This paper presents an analysis for determining when a bounded number of processes and channels will be generated. The analysis proceeds in two stages. First we extend a polymorphic type system for SML to deduce not only the type of CML programs but also their communication behaviour expressed as terms in a new process algebra. Next we develop an analysis that given

1 Introduction

Higher-order concurrent languages as CML [8] and Facile [2] offer mechanisms for the *dynamic* creation of channels and processes in addition to the possibility of sending and receiving values over channels. To obtain an *efficient implementation* of programs in such languages we would need information about their communication topology:

- Does the program only spawn a finite number of processes?
- Does the program only create a finite number of channels?

If the answer to the first question is yes we may load the processes on the available processors and dispense with

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- *Parallel control-flow*

$$(\textsc{fork})\ \frac{\Gamma \vdash e : \mathsf{unit} \xrightarrow{\Phi_0} \tau, \epsilon}{\Gamma \vdash \mathit{fork}\, e : \tau,\ \mathit{FORK}\, \Phi_0}$$

…in 1999, rich general effect-system structure

Type and Effect Systems

Flemming Nielson and Hanne Riis Nielson

Department of Computer Science, Aarhus University, Denmark.

Abstract. The design and implementation of a correct system can benefit from employing static techniques for ensuring that the dynamic behaviour satisfies the specification. Many programming languages incorporate types for ensuring that certain operations are only applied to data of the appropriate form. A natural extension of type checking techniques is to enrich the types with annotations and effects that further describe intensional aspects of the dynamic behaviour.

Keywords. Polymorphic type systems, effect annotations, subeffecting and subtyping, semantic correctness, type inference algorithms, syntactic soundness and completeness. Analyses for control flow, binding times, side effects, region structure, and communication structure.

1 Introduction

Static analysis of programs comprises a broad collection of techniques for predicting safe and computable approximations to the set of values or behaviours arising dynamically during computation; this may be used to validate program transformations, to generate more efficient code or to increase the understanding

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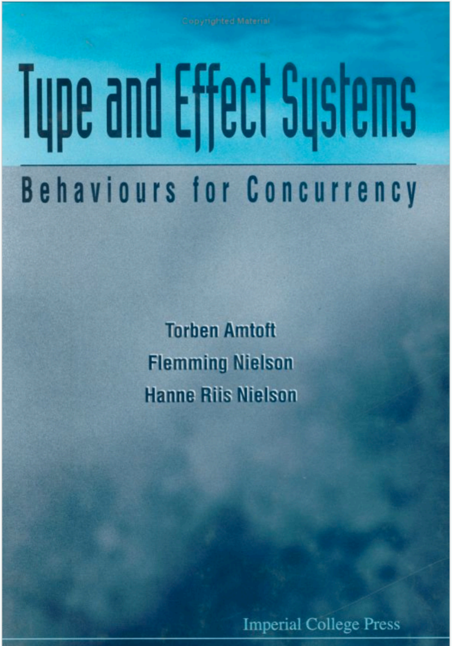

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- ... with fixed-point $\mathsf{rec}\beta.\Phi$
- ... subeffecting axioms provide semiring equations, *e.g.*

$$(\Phi_1 + \Phi_2); \Phi_3 \sqsubseteq (\Phi_1; \Phi_3) + (\Phi_1; \Phi_2)$$
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For our musical example...

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- **Song structure** via term-algebra:

$$\Phi = \mathsf{C}, \mathsf{D}, \mathsf{E}, \ldots, \mathsf{rest} \mid \Phi_1 + \Phi_2 \mid \Phi_1 \bullet \Phi_2 \mid \Phi^*$$

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  gives 13 seconds

from "Type and effect systems" (Nielson, Nielson, 1999)

"Now we turn to explaining the individual steps in the overall methodology of designing and using type and effect systems:

- *devise a semantics for the programming language,*
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- An effect-directed semantics unifies the first three steps
- We develop an effect-directed semantics for rich Nielson-Nielson-style effects

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Core idea: algebra-semantics homomorphism

Algebraic structure of $\mathcal{F}$ determines structure on family $D_\tau^{\mathcal{F}}$.

Effect analysis and semantics

Lattice effects
(Gifford, Lucassen '86)
$\Gamma \vdash e : \tau, \mathcal{F}$

⟹

Richer structure on $\mathcal{F}$
(Nielson, Nielson '94, '99)

⇓

"Marriage of effects and monads"
(Wadler, Thiemann '03)
$[\![\Gamma \vdash e : M^{\mathcal{F}}\tau]\!] : [\![\Gamma]\!] \to \mathsf{T}[\![\tau]\!]$

⇓

Effect-directed semantics
(Katsumata '14)
$[\![\Gamma \vdash e : M^{\mathcal{F}}\tau]\!] : [\![\Gamma]\!] \to \mathsf{T}_{\mathcal{F}}[\![\tau]\!]$
(graded monads)

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This paper
Richer effect-directed semantics
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Operations on $\mathsf{T}_{\mathcal{F}}$ homomorphic to operations on $\mathcal{F}$

Modelling effects with graded monads (Katsumata 2014)

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Given $d_1 : A \to \mathsf{T}_F B$ and $d_2 : B \to \mathsf{T}_G C$

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Modelling effects with graded monads (Katsumata 2014)

$$[\![\Gamma \vdash e : \tau, F]\!] : [\![\Gamma]\!] \rightarrow \mathsf{T}_F [\![\tau]\!]$$

e.g. for state $\mathsf{T}_F A = (\mathit{reads}(F)) \rightarrow (A \times \mathit{writes}(F))$.
e.g. for partiality $\mathsf{T}_\bot A = \bot$, $\mathsf{T}_\top A = A$, $\mathsf{T}_? A = A + 1$

Graded monads provide sequential composition

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With ordering

Given partially ordered $(\mathcal{F}, \bullet, 0, \sqsubseteq)$ then for all $F \sqsubseteq G$ then coercion: $\iota_{F,G,A} : \mathsf{T}_F A \rightarrow \mathsf{T}_G A$

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- Instead: parameterise semantics on
  $\mathrm{COND} : \mathsf{T}_F \mathbb{B} \times \mathsf{T}_G A \times \mathsf{T}_H A \to \mathsf{T}_{?+(F,G,H)} A$

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- all operations are monotonic with respect to $\sqsubseteq$.

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Monoids are to (graded) monads as jonoids are to (graded) joinads

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\begin{array}{rl}
(\text{IF}\beta 1') & \mathsf{if\,true\,then}\; e \;\mathsf{else}\; x \equiv e \\
(\text{IF}\beta 2') & \mathsf{if\,false\,then}\; x \;\mathsf{else}\; e' \equiv e' \\
(\text{IF-DIST-PAR}) & (\mathsf{if}\; b \;\mathsf{then}\; e \;\mathsf{else}\; e')\;\mathsf{par}\; e'' \\
& \quad \equiv \mathsf{if}\; b \;\mathsf{then}\; (e \;\mathsf{par}\; e'') \;\mathsf{else}\; (e' \;\mathsf{par}\; e'') \\
(\text{IF-DIST-SEQ}) & \mathsf{let}\; x = (\mathsf{if}\; e \;\mathsf{then}\; e' \;\mathsf{else}\; e'') \;\mathsf{in}\; e''' \\
& \quad \equiv \mathsf{if}\; e \;\mathsf{then}\; (\mathsf{let}\; x = e' \;\mathsf{in}\; e''') \;\mathsf{else}\; (\mathsf{let}\; x = e'' \;\mathsf{in}\; e''') \\
(\text{PAR-PURE}) & x \;\mathsf{par}\; e \equiv (x, e) \\
(\text{PAR-SYM}) & e \;\mathsf{par}\; e' \equiv \mathsf{swap}\; (e' \;\mathsf{par}\; e) \\
(\text{PAR-ASSOC}) & e \;\mathsf{par}\; (e' \;\mathsf{par}\; e'') \equiv \mathsf{assoc}\; ((e \;\mathsf{par}\; e') \;\mathsf{par}\; e'')
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  - Exposes which structure is needed in each direction

Thanks Hanne and Flemming for the inspiration.
Happy Birthday!

Thanks to Sam Aaron (Cambridge) for the Sonic Pi language used for the intro program

Backup slides

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