Formal Treatment of Non-accumulation

The maths here is associated with the “limits of evidence accumulation” work (Avilés, Bowman & Wyble, 2020; Bowman & Avilés, under submission). It verifies a formulation of our probability of first seeing idea given by the first reviewer of (Bowman & Avilés, under submission).

Assume that p is the probability of consciously perceiving the repeating stimulus at an arbitrary presentation, if it has not yet been perceived. Additionally, we assume that once the repeating stimulus has been consciously perceived for the first time, repetitions of it are perceived with a probability of one.

We call the repeating stimulus the target, even though its identity is not pre-specified.

The cumulative probability that the target has been consciously perceived at least once by a presentation i, is denoted $cp(i)$ and defined as follows for specific values of i,

$$cp(1) = p$$

$$cp(2) = p + (1-p).p$$

$$cp(3) = p + (1-p).(p + (1-p).p) = p + (1-p).p + (1-p)^2.p$$

$cp(1)$ is straightforward. $cp(2)$ reflects the fact that the target could have been perceived on the first presentation, and if it was not (i.e. $(1-p)$), then it could have been perceived on the second presentation. $cp(3)$ continues with this logic. In addition, this cumulative probability can be defined fully generally (i.e. for an arbitrary i) as the following recursive equation,

$$cp(i+1) = cp(i) + (1-p)^i.p$$

or analogously as,

$$cp(i+1) = p + (1-p).p + (1-p)^2.p + \cdots + (1-p)^i.p = \sum_{k=0}^{i}(1-p)^k.p$$

It can be shown that $cp(i+1)$ approaches 1 as i increases, but never exceeds it.

We can also define the proportion of trials for which the target has not been consciously perceived by presentation i, denoted $np(i)$, as follows,

$$np(i) = 1 - cp(i)$$

The quantity that we are interested in is the probability of seeing a repetition for the first time, expressed as a conditional probability. That is, the probability of seeing a repetition on trial $i+1$, given that it has not been previously seen, i.e. on the previous i trials. We denote this as $P_{fst}(i+1)$, which, using the notation in appendix A of Avilés, Bowman & Wyble (2020), can be stated as,

$$P_{fst}(i+1) = p(\,See\_Repeating(i+1) \mid \forall j \in \mathbb{N}\ (1 \le j \le i\,) \cdot \neg\, See\_Repeating(j)\,)$$

and (using that $P(A|B) = P(A \wedge B)/P(B)$), we define this probability as follows,

$$P_{fst}(i+1) = \frac{cp(i+1) - cp(i)}{np(i)}$$

The key proposition that underlies our claims is the following.

Proposition (constant first probability)

If p is constant across presentations then,

$$\forall N\ .\ P_{fst}(i+1) = \frac{cp(i+1) - cp(i)}{np(i)} = p$$

Proof

We proceed by induction.

Base case

Assume that $i = 1$. Then, by substituting definitions and cancelling, we obtain the following:

$$P_{fst}(2) = \frac{cp(2) - cp(1)}{np(1)} = \frac{(p + (1-p).p) - p}{1-p} = \frac{(1-p).p}{1-p} = p$$

Although not required for the induction proof, we also verify the $i = 2$ case in order to see the form of the proof for the more complex induction case.

Assume that $i = 2$. Then, by substituting definitions, cancelling and factorisation, we obtain the following:

$$\begin{aligned} P_{fst}(3) = \frac{cp(3) - cp(2)}{np(2)} &= \frac{\big(p + (1-p).(p + (1-p).p)\big) - (p + (1-p).p)}{1 - (p + (1-p).p)} \\ &= \frac{(1-p).p + (1-p)^2.p - (1-p).p}{1 - p - (1-p).p} = \frac{(1-p).(1-p).p}{(1-p) - (1-p).p} \\ &= \frac{(1-p).(1-p).p}{(1-p).(1-p)} = p \end{aligned}$$

Induction step

Here, we assume the property for i repetitions, i.e.

$$P_{fst}(i) = \frac{cp(i) - cp(i-1)}{np(i-1)} = p$$

which we will use as,

$$cp(i) - cp(i-1) = p.np(i-1) \qquad \text{(eqn *)}$$

We seek to show the property for $i + 1$, i.e.

$$P_{fst}(i+1) = \frac{cp(i+1) - cp(i)}{np(i)} = p$$

we proceed using substitution, manipulation and cancellation as follows,

$$\begin{aligned} P_{fst}(i+1) = \frac{cp(i+1) - cp(i)}{np(i)} &= \frac{(cp(i) + (1-p)^i p) - (cp(i-1) + (1-p)^{(i-1)}p)}{1 - (cp(i-1) + (1-p)^{(i-1)}p)} \\ &= \frac{cp(i) - cp(i-1) + (1-p)^i p - (1-p)^{(i-1)}p}{1 - cp(i-1) - (1-p)^{(i-1)}p} \\ &= \frac{cp(i) - cp(i-1) + (1-p)^{(i-1)}p((1-p) - 1)}{1 - cp(i-1) - (1-p)^{(i-1)}p} \\ &= \frac{cp(i) - cp(i-1) - (1-p)^{(i-1)}p^2}{1 - cp(i-1) - (1-p)^{(i-1)}p} \end{aligned}$$

We now use equation (*), the definition of np, factorisation and cancellation to derive the following:

$$\frac{cp(i) - cp(i-1) - (1-p)^{(i-1)}p^2}{1 - cp(i-1) - (1-p)^{(i-1)}p} = \frac{p.np(i-1) - (1-p)^{(i-1)}p^2}{np(i-1) - (1-p)^{(i-1)}p}$$
$$= \frac{p.(np(i-1) - (1-p)^{(i-1)}p)}{(np(i-1) - (1-p)^{(i-1)}p)} = p$$

Therefore, we have shown that, if we assume that $P_{fst}(i) = p$, we can derive that $P_{fst}(i+1) = p$. From this, since $P_{fst}(2) = p$ (the base case), we can deduce by induction that $P_{fst}(i+1) = p$ for all i. QED

Avilés, A., Bowman, H., & Wyble, B. (2020). On the limits of evidence accumulation of the preconscious percept. Cognition, 195, 104080.

Bowman, H., & Avilés, A. (under submission). No subliminal memory for spaced repeated images in rapid-serial-visual-presentation streams of thousands of images