Debugging Functional Programs

Olaf Chitil
Partially supported by EPSRC grant EP/C516605/1

University of Kent

April 2009

A Faulty Haskell Program

```
main = putStrLn (sort "sort")

sort :: Ord a => [a] -> [a]

sort [] = []
sort (x:xs) = insert x (sort xs)

insert :: Ord a => a -> [a] -> [a]

insert x [] = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x : ys
```

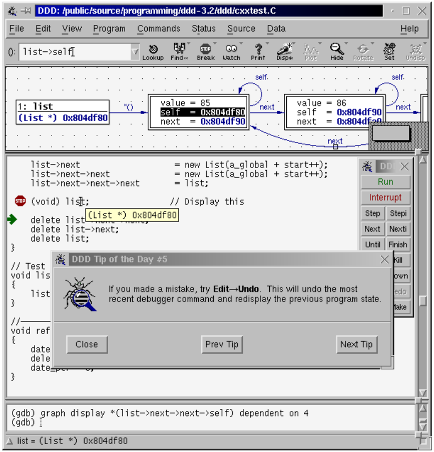
Output: `os`

Observable faulty behaviour:

- wrong result
- abortion with run-time error
- non-termination

Conventional Debugging Methods

- The print / logging method: Add print statements to program.
- A stepping debugger such as the Data Display Debugger (DDD)



Why Debug Functional Programs Differently?

- Conventional methods are ill-suited for non-strict functional languages.
- New, more powerful methods can take advantage of properties of purely functional languages.

Haskell: A Non-Strict Purely Functional Programming Language

- Non-strict function: it has a well-defined result even when (parts of) arguments are unknown or ill-defined.
- Purely functional: an expression only denotes a value, no state transformation.

Properties:

- Rich but simple equational program algebra.

  ```
  map f . map g = map (f . g)
  ```

- Can evaluate function arguments in any order (or not at all).

  ```
  f (g 3 4) (h 1 2) (i 5 (j 3 9) (k 4))
  ```

- Enables programming with recursive values, infinite data structures and efficient data-oriented programming.

  ```
  pExp = pChar '(' >> pExp >> pChar '+' >> pExp >> pChar ')'
  ```

  ```
  factorial n = product [1..n]
  ```

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

elem 42 [1..]

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

$\underline{\texttt{elem 42 [1..]}}$
$\leadsto$ `or (map (== 42)` $\underline{\texttt{[1..]}}$`)`

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

$\underline{\texttt{elem 42 [1..]}}$
$\rightsquigarrow$ `or (map (== 42)` $\underline{\texttt{[1..]}}$`)`
$\rightsquigarrow$ `or (`$\underline{\texttt{map (== 42) (1:[2..])}}$`)`

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

```
elem 42 [1..]
 ⇝ or (map (== 42) [1..])
 ⇝ or (map (== 42) (1:[2..]))
 ⇝ or (False : map (== 42) [2..])
```

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

```
elem 42 [1..]
 ⇝ or (map (== 42) [1..])
 ⇝ or (map (== 42) (1:[2..]))
 ⇝ or (False : map (== 42) [2..])
 ⇝ or (map (== 42) [2..])
```

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

```
elem 42 [1..]
 ⇝ or (map (== 42) [1..])
 ⇝ or (map (== 42) (1:[2..]))
 ⇝ or (False : map (== 42) [2..])
 ⇝ or (map (== 42) [2..])
 ⇝ or (map (== 42) (2:[3..]))
```

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

$$
\begin{array}{l}
\underline{\texttt{elem 42 [1..]}} \\
\leadsto \texttt{or (map (== 42) } \underline{\texttt{[1..]}}\texttt{)} \\
\leadsto \texttt{or (}\underline{\texttt{map (== 42) (1:[2..])}}\texttt{)} \\
\leadsto \underline{\texttt{or (False : map (== 42) [2..])}} \\
\leadsto \texttt{or (map (== 42) } \underline{\texttt{[2..]}}\texttt{)} \\
\leadsto \texttt{or (}\underline{\texttt{map (== 42) (2:[3..])}}\texttt{)} \\
\leadsto \underline{\texttt{or (False : map (== 42) [3..])}}
\end{array}
$$

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

```
elem 42 [1..]
 ⇝ or (map (== 42) [1..])
 ⇝ or (map (== 42) (1:[2..]))
 ⇝ or (False : map (== 42) [2..])
 ⇝ or (map (== 42) [2..])
 ⇝ or (map (== 42) (2:[3..]))
 ⇝ or (False : map (== 42) [3..])
 ⇝ or (map (== 42) [3..])
```

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

```
elem 42 [1..]
 ⇝ or (map (== 42) [1..])
 ⇝ or (map (== 42) (1:[2..]))
 ⇝ or (False : map (== 42) [2..])
 ⇝ or (map (== 42) [2..])
 ⇝ or (map (== 42) (2:[3..]))
 ⇝ or (False : map (== 42) [3..])
 ⇝ or (map (== 42) [3..])
 ⋮   ⋮
```

Evaluation of an expression

```
elem :: Int -> [Int] -> Bool
elem x xs = or (map (==x) xs)
```

```
elem 42 [1..]
 ⇝ or (map (== 42) [1..])
 ⇝ or (map (== 42) (1:[2..]))
 ⇝ or (False : map (== 42) [2..])
 ⇝ or (map (== 42) [2..])
 ⇝ or (map (== 42) (2:[3..]))
 ⇝ or (False : map (== 42) [3..])
 ⇝ or (map (== 42) [3..])
 ⋮   ⋮
 ⇝ True
```

Here reduction steps for `map` and `or` are skipped.

Why stepping doesn't work

- No stepping through sequence of statements in source code.
- Complex evaluation order.
- Run-time stack unrelated to static function call structure.
- Unevaluated subexpressions large and hard to read.

Why Printing doesn't work

Impure function `traceShow :: String -> [Int] -> [Int]`

```
insert :: Int -> [Int] -> [Int]

insert x [] = [x]
insert x (y:ys) =
  if x > y then y : (traceShow ">" (insert x ys))
           else x:y:ys

main = print (take 5 (insert 4 [1..]))
```

Output:

```
[1>[2>[3>[4,4,5,6,7,8,9,10,11,...
```

- output mixed up
- non-termination ⇒ observation changes behaviour

Properties of Functional Languages

- No canonical execution model.
  - various reduction semantics (small step, big step)
  - interpreters with environments (explicit substitutions)
  - also denotational semantics
- An expression denotes only a value
  - independent evaluation of subexpressions

    ```
    f (g 3 4) (h 1 2) (i 5) (j 3 9 3)
    ```

Properties of Functional Languages

- No canonical execution model.
  - various reduction semantics (small step, big step)
  - interpreters with environments (explicit substitutions)
  - also denotational semantics
- An expression denotes only a value
  - independent evaluation of subexpressions

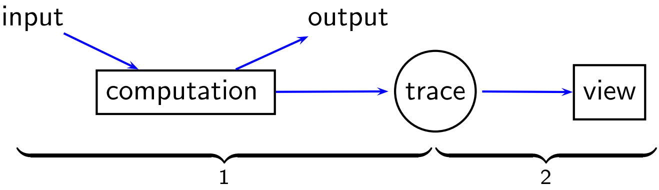
    ```
    f (g 3 4) (h 1 2) (i 5) (j 3 9 3)
    ```

Advantages for Debugging

- Many semantic models as potential basis.
- Simple and compositional semantics.
- Freedom from sequentiality of computation.

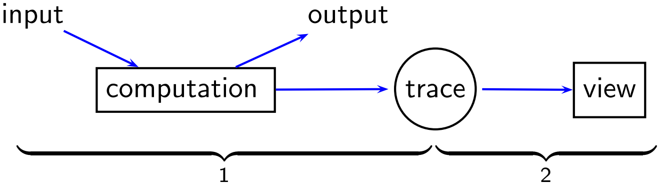
Outline

Two-Phase Tracing



Liberates from time arrow of computation.

Two-Phase Tracing

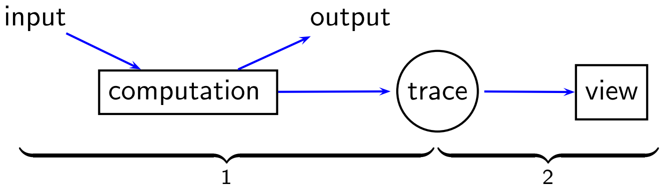


Liberates from time arrow of computation.

Trace stored in

- Memory.
- File.
- Generated on demand by reexecution.

Two-Phase Tracing



Liberates from time arrow of computation.

Trace stored in

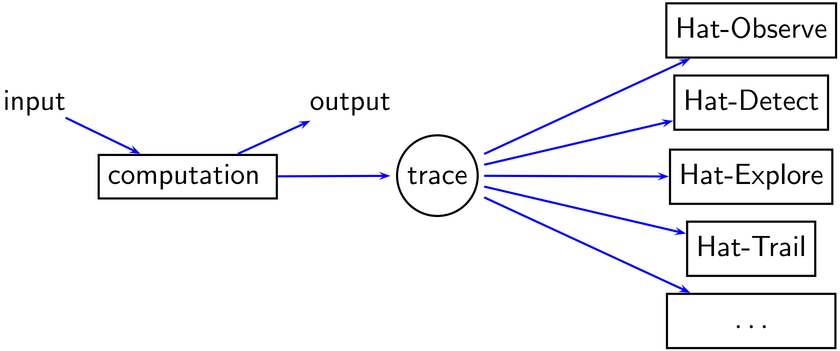
- Memory.
- File.
- Generated on demand by reexecution.

Trace Generation

- Program annotations + library.
- Program transformation.
- Modified abstract machine.

Hat

- Multi-View Tracer



- For Haskell 98 + some extensions.
- Developed by Colin Runciman, Jan Sparud, Malcolm Wallace, Olaf Chitil, Thorsten Brehm, Tom Davie, Tom Shackell, ...

Faulty Insertion Sort

```
main = putStrLn (sort "sort")

sort :: Ord a => [a] -> [a]
sort [] = []
sort (x:xs) = insert x (sort xs)

insert :: Ord a => a -> [a] -> [a]
insert x [] = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x : ys
```

Output:

```
os
```

Observation of Expressions and Functions

Observation of Expressions and Functions

Observation of function sort:

```
sort "sort" = "os"
sort "ort" = "o"
sort "rt" = "r"
sort "t" = "t"
sort "" = ""
```

Observation of function insert:

```
insert 's' "o" = "os"
insert 's' "" = "s"
insert 'o' "r" = "o"
insert 'r' "t" = "r"
insert 't' "" = "t"
```

Observation of Expressions and Functions

- Haskell Object Observation Debugger (Hood) by Andy Gill.
  - A library.
  - Programmer annotates expressions of interest.
  - Annotated expressions are traced during computation.
  - The print method for the lazy functional programmer.
- Observation of functions most useful.
- Relates to denotational semantics.

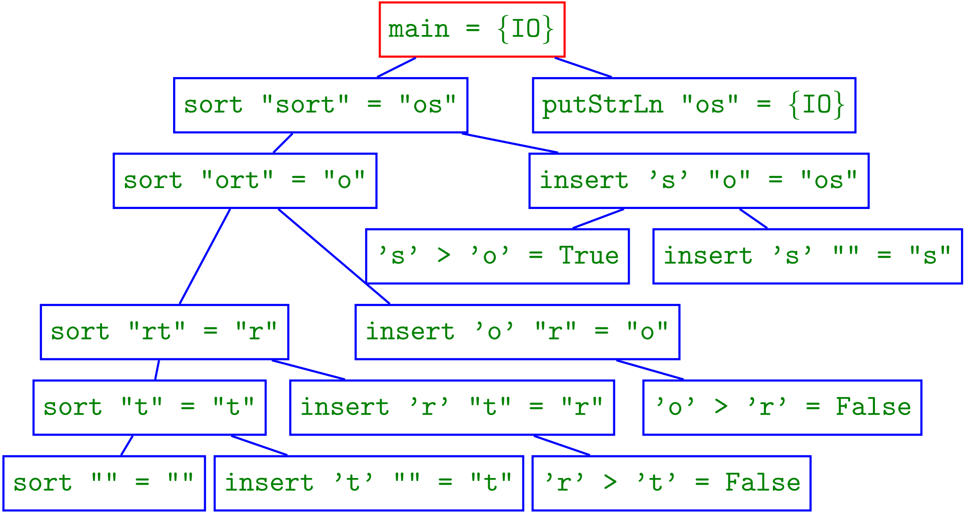
```
insert 3 (1:2:3:4:_) = 1:2:3:4:_
insert 3 (2:3:4:_) = 2:3:4:_
insert 3 (3:4:_) = 3:4:_
```

Algorithmic Debugging

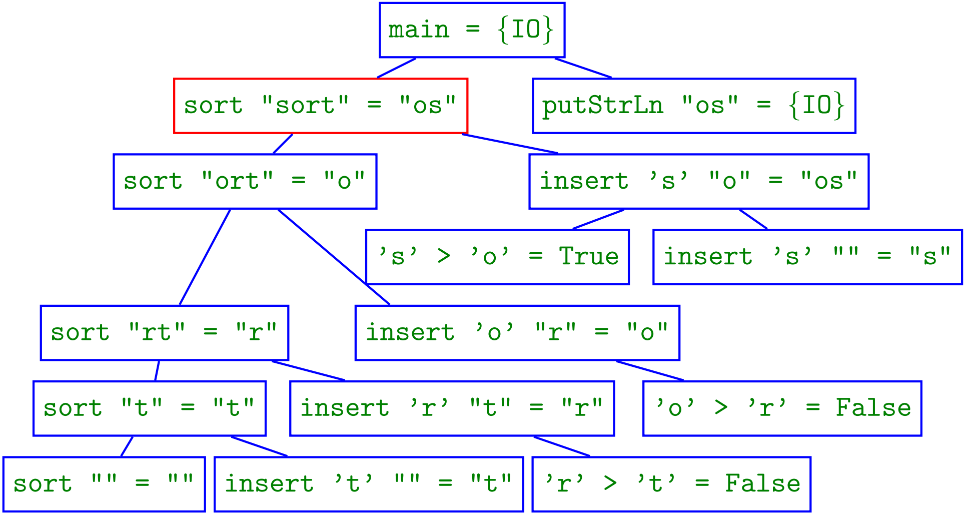
Algorithmic Debugging

```
sort "sort" = "os" ?
n
insert 's' "o" = "os" ?
y
sort "ort" = "o" ?
n
insert 'o' "r" = "o" ?
n
Bug identified:
  "Insert.hs":8-9:
  insert x [] = [x]
  insert x (y:ys) = if x > y then y:(insert x ys) else x:ys
```

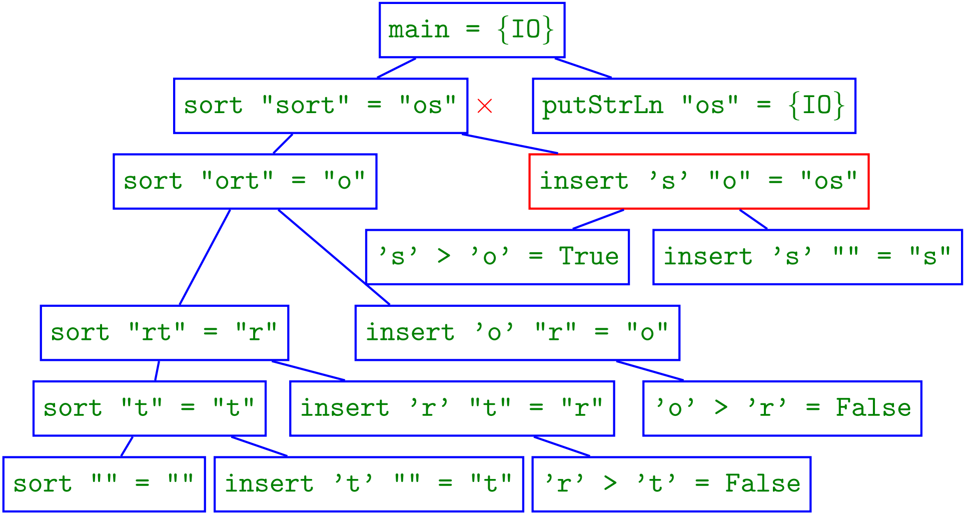
The Computation Tree



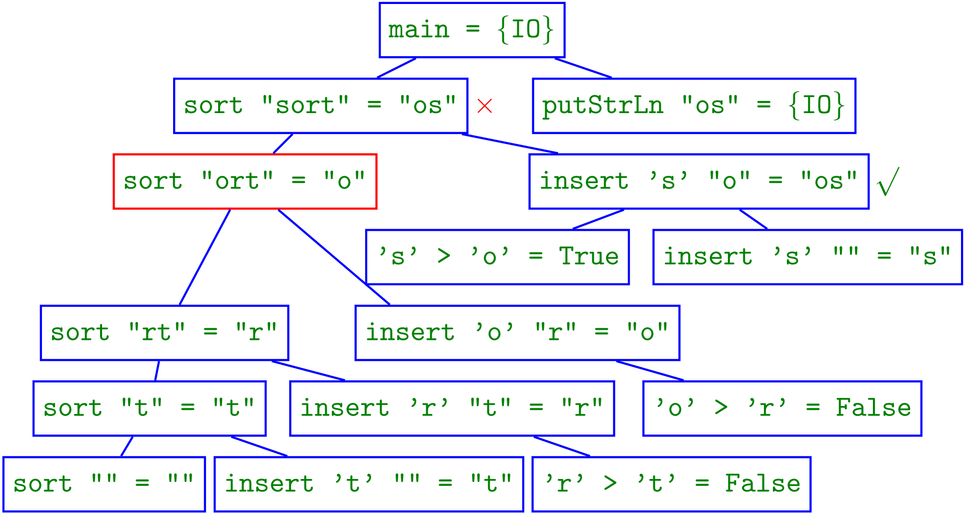
The Computation Tree



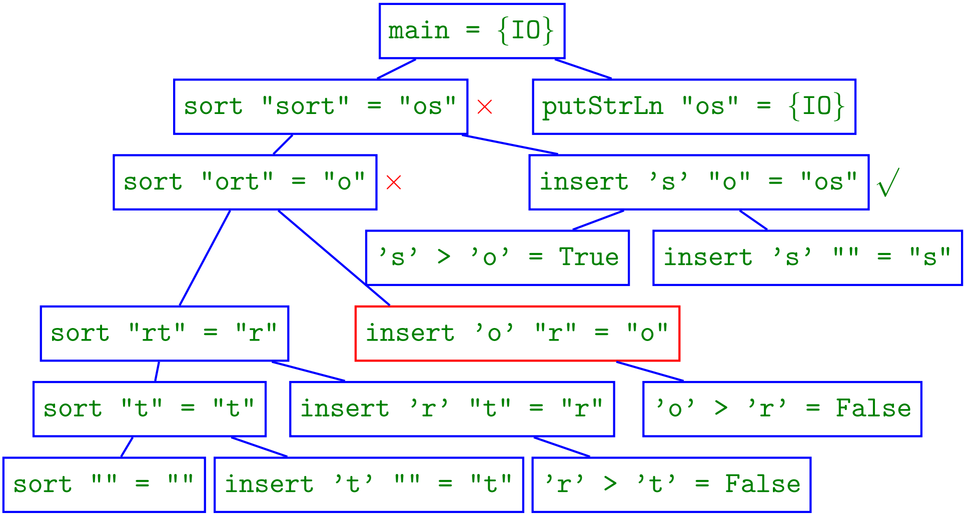
The Computation Tree



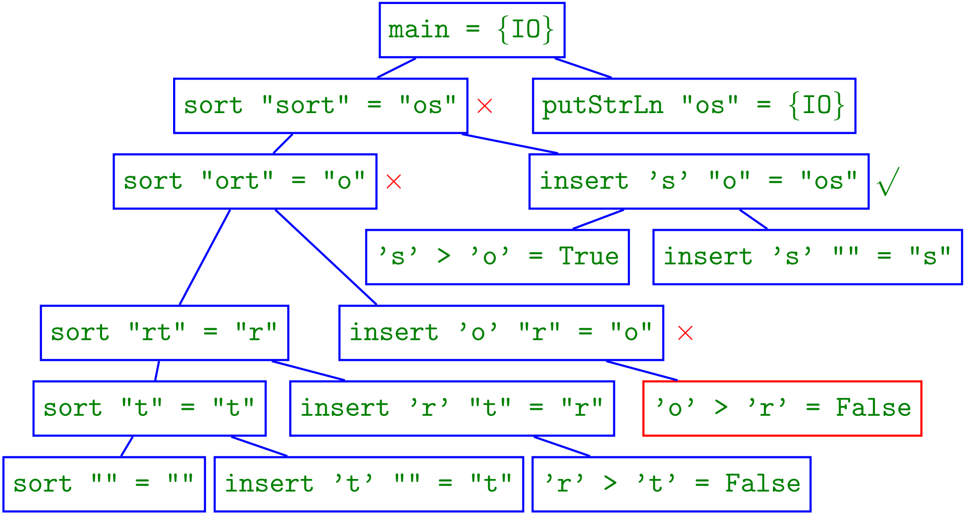
The Computation Tree



The Computation Tree



The Computation Tree



Fault located!

```
main = putStrLn (sort "sort")

sort :: Ord a => [a] -> [a]
sort []     = []
sort (x:xs) = insert x (sort xs)

insert :: Ord a => a -> [a] -> [a]
insert x []     = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x:ys
```

Faulty computation: `insert 'o' "r" = "o"`

Algorithmic Debugging

- Shapiro for Prolog, 1983.
- Henrik Nilsson's Freija for lazy functional language, 1998.
- Bernie Pope's Buddha for Haskell, 2003.

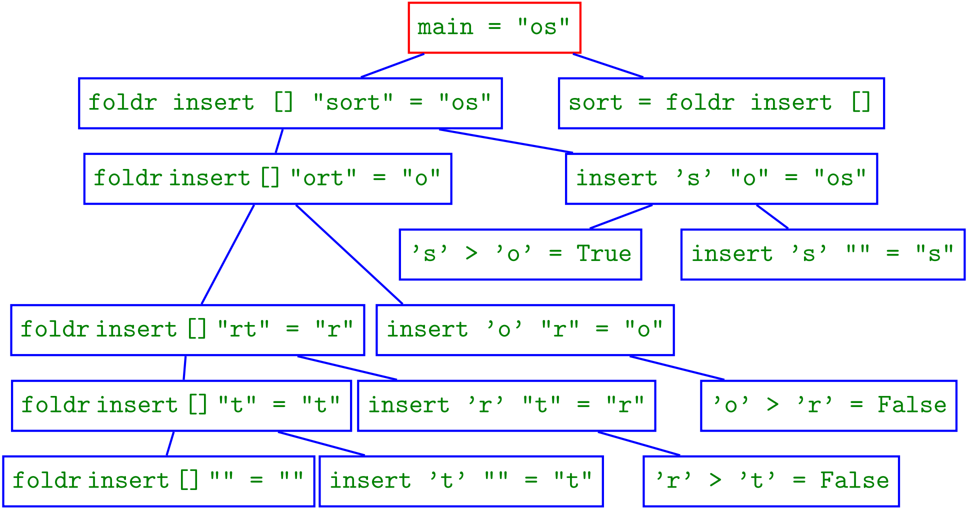
- Correctness of tree node according to intended semantics.
- Incorrect node whose children are all correct is faulty.
- Each node relates to (part of) a function definition.

- Relates to natural, big-step semantics.

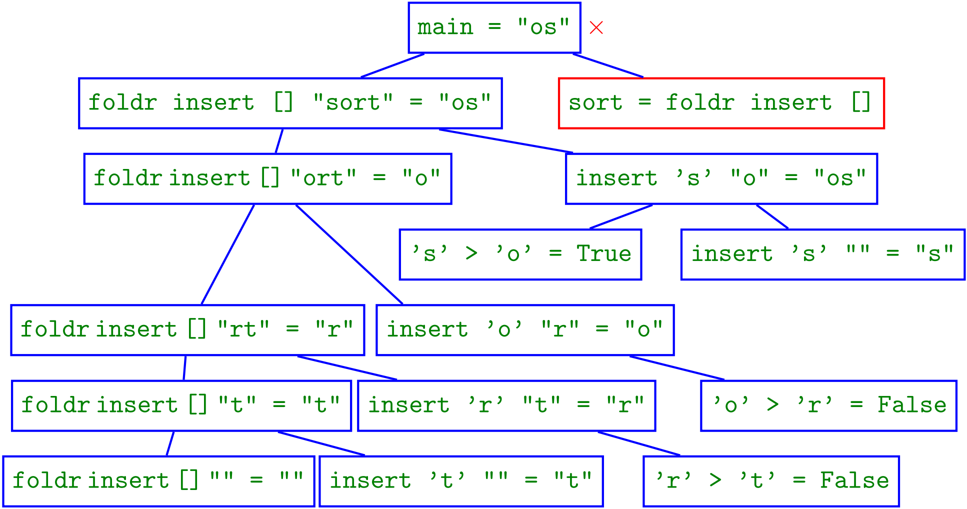
Higher-Order Insertion Sort

```
main :: String
main = sort "sort"

sort :: Ord a => [a] -> [a]
sort = foldr insert []

foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f a []     = a
foldr f a (x:xs) = f x (foldr f a xs)

insert :: Ord a => a -> [a] -> [a]
insert x []     = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x:ys
```

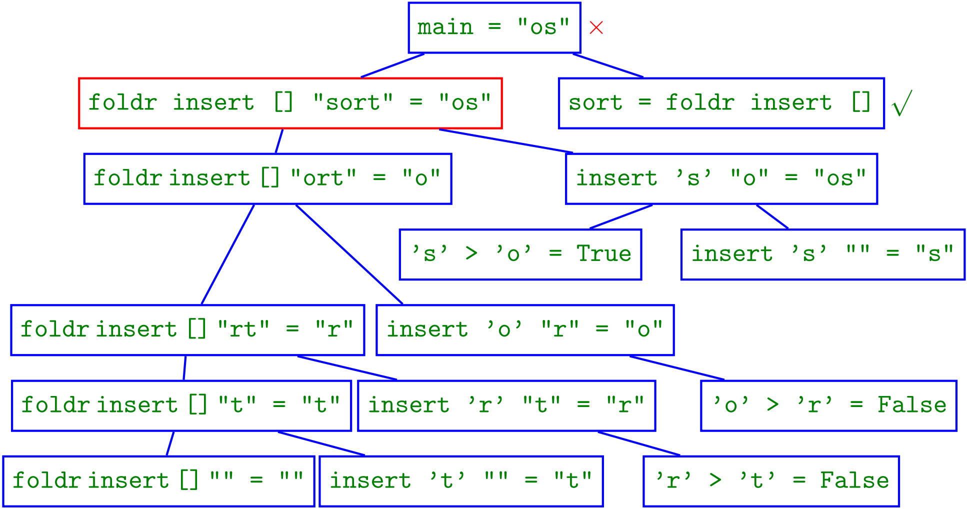
Higher-Order Algorithmic Debugging



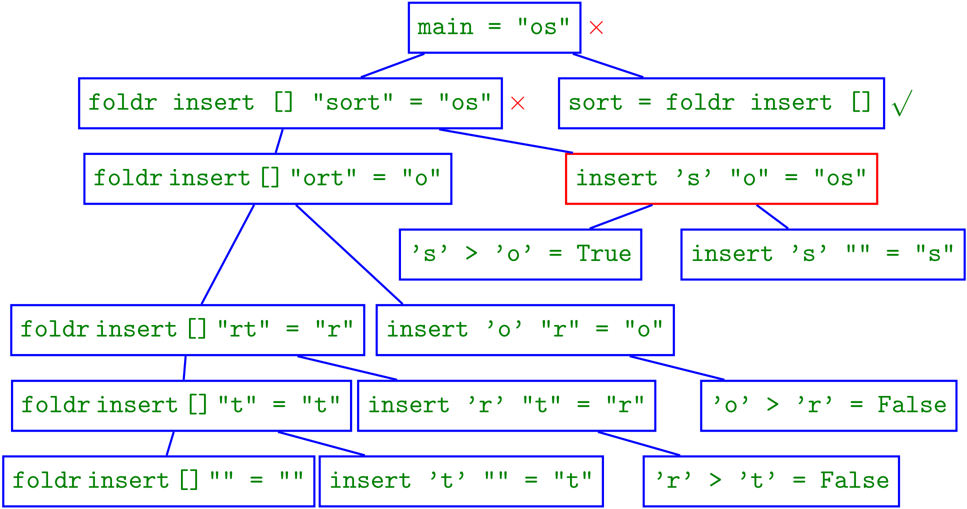
Higher-Order Algorithmic Debugging



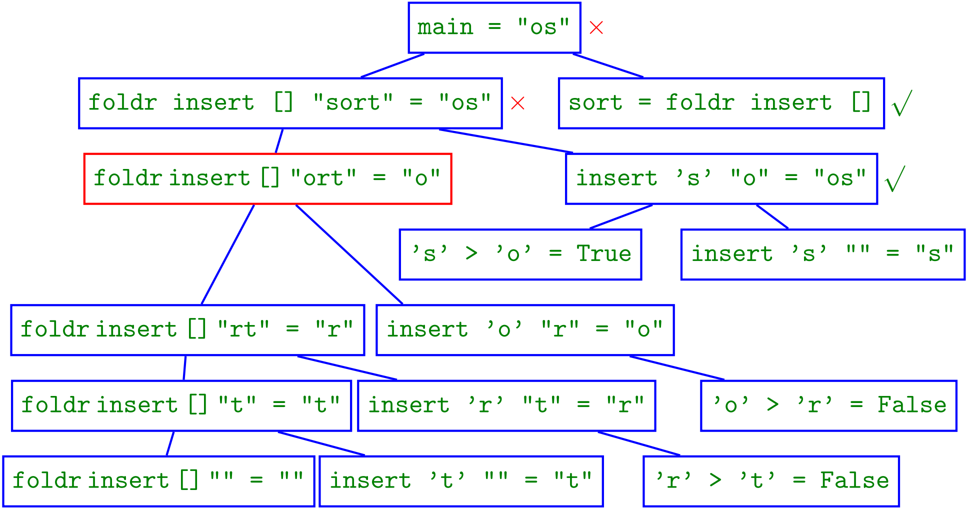
Higher-Order Algorithmic Debugging



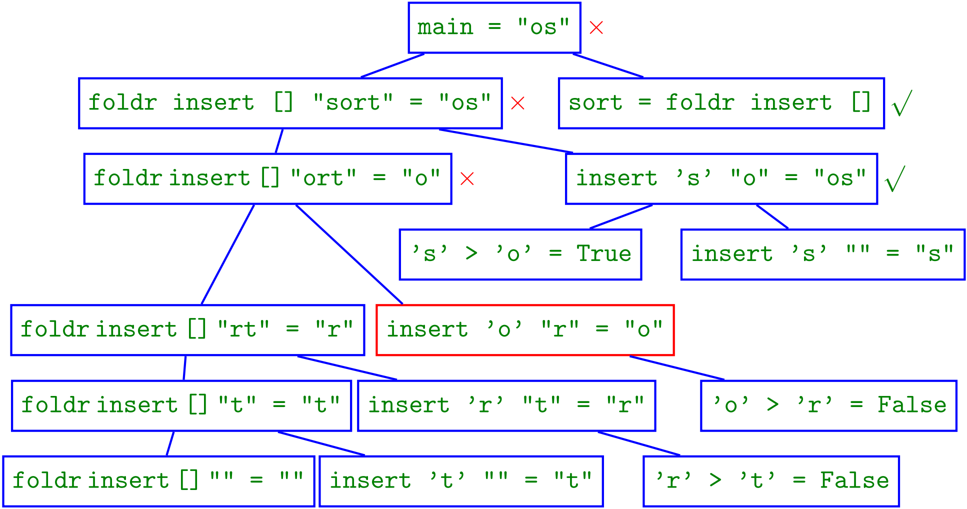
Higher-Order Algorithmic Debugging

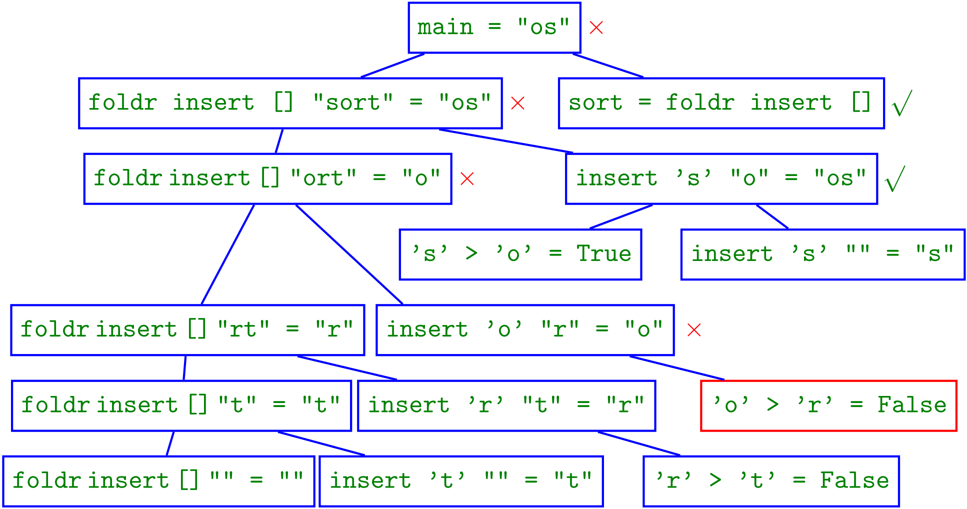


Higher-Order Algorithmic Debugging

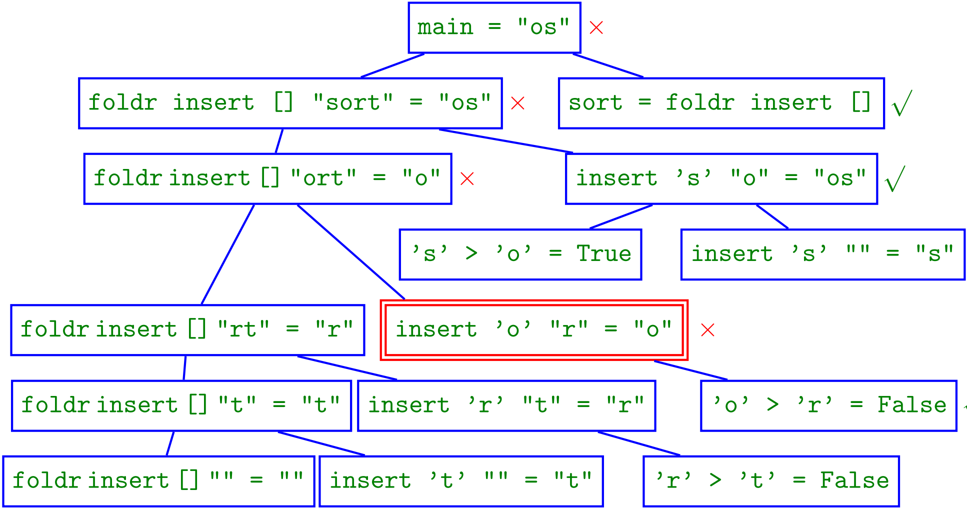


Higher-Order Algorithmic Debugging

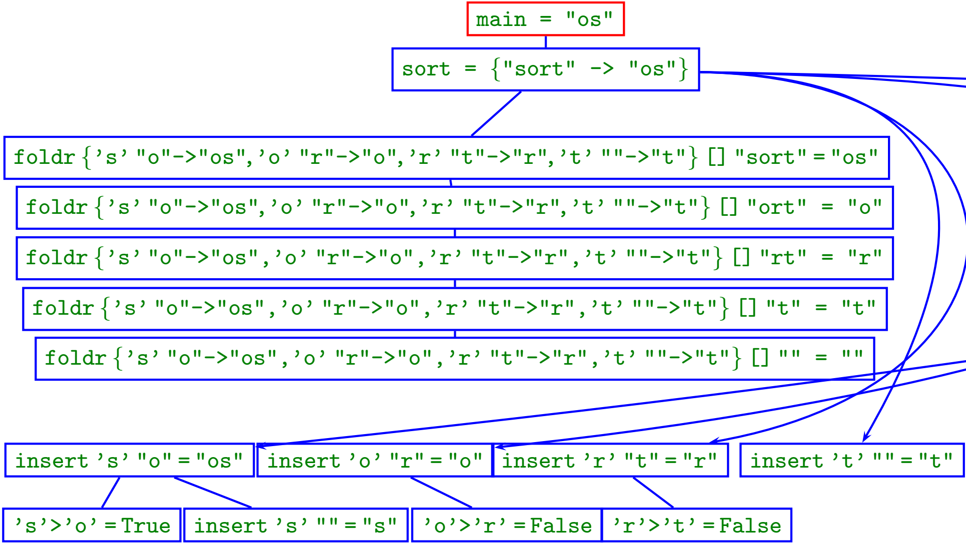



main = "os" ×
foldr insert [] "sort" = "os" ×
sort = foldr insert [] √
foldr insert [] "ort" = "o" ×
insert 's' "o" = "os" √
's' > 'o' = True
insert 's' "" = "s"
foldr insert [] "rt" = "r"
insert 'o' "r" = "o" ×
foldr insert [] "t" = "t"
insert 'r' "t" = "r"
'o' > 'r' = False
foldr insert [] "" = ""
insert 't' "" = "t"
'r' > 't' = False

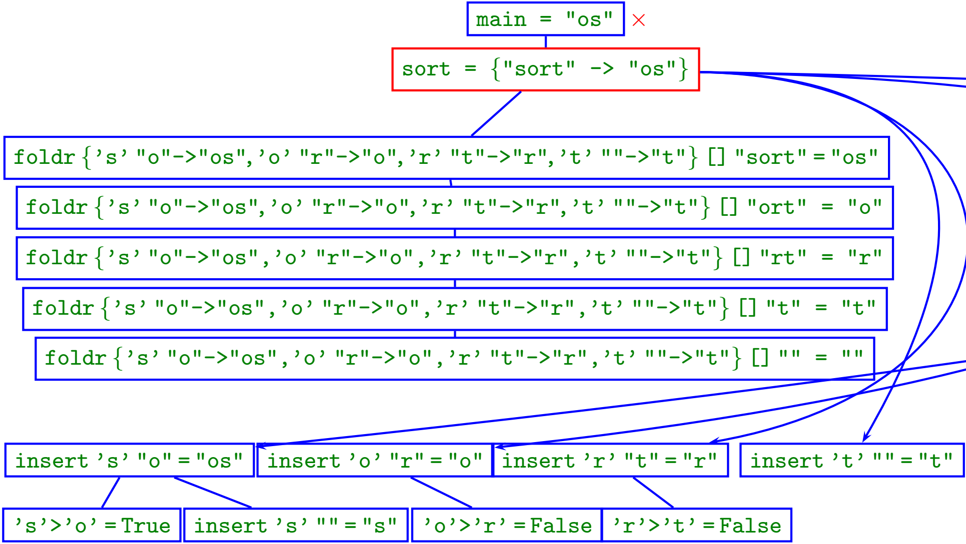
Higher-Order Algorithmic Debugging



Higher-Order Algorithmic Debugging II



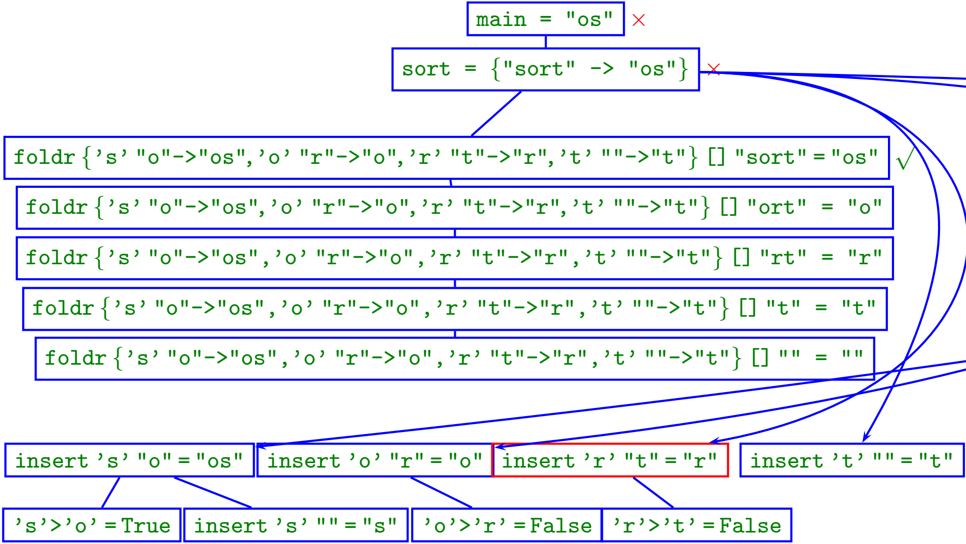
Higher-Order Algorithmic Debugging II



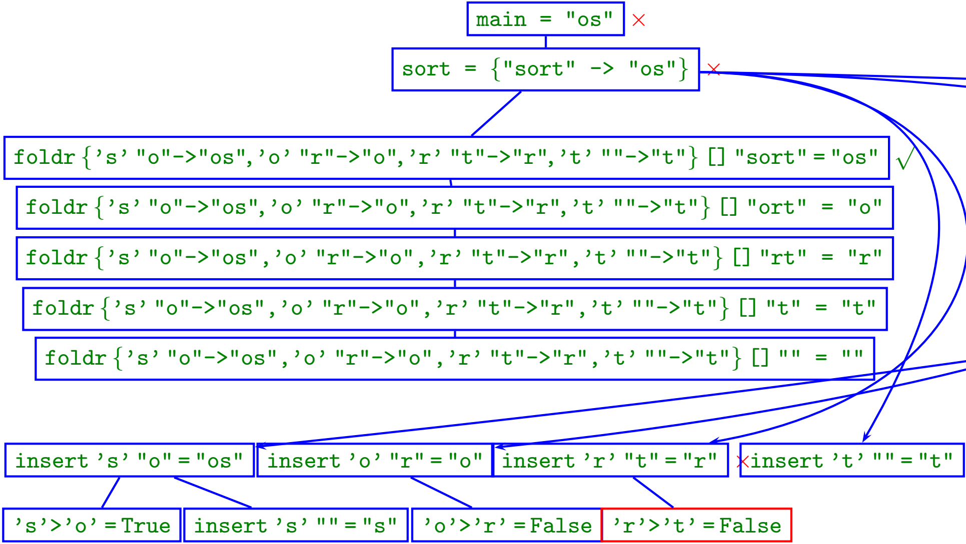
Higher-Order Algorithmic Debugging II

```
main = "os"
sort = {"sort" -> "os"}
foldr {'s' "o"->"os",'o' "r"->"o",'r' "t"->"r",'t' ""->"t"} [] "sort" = "os"
foldr {'s' "o"->"os",'o' "r"->"o",'r' "t"->"r",'t' ""->"t"} [] "ort" = "o"
foldr {'s' "o"->"os",'o' "r"->"o",'r' "t"->"r",'t' ""->"t"} [] "rt" = "r"
foldr {'s' "o"->"os",'o' "r"->"o",'r' "t"->"r",'t' ""->"t"} [] "t" = "t"
foldr {'s' "o"->"os",'o' "r"->"o",'r' "t"->"r",'t' ""->"t"} [] "" = ""
insert 's' "o" = "os"    insert 'o' "r" = "o"    insert 'r' "t" = "r"    insert 't' "" = "t"
's'>'o' = True    insert 's' "" = "s"    'o'>'r' = False    'r'>'t' = False
```

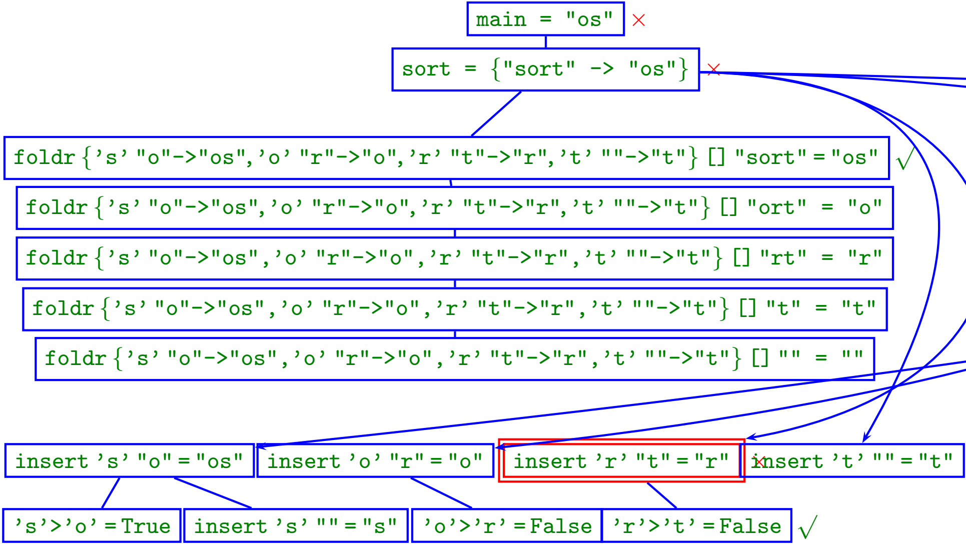
Higher-Order Algorithmic Debugging II



Higher-Order Algorithmic Debugging II



Higher-Order Algorithmic Debugging II



Source-based Free Navigation and Program Slicing

Source-based Free Navigation and Program Slicing

```
==== Hat-Explore 2.00 ==== Call 2/2 =======================
 1.  main = {IO}
 2.  sort "sort" = "os"
 3.  sort "ort" = "o"

---- Insert.hs ---- lines 5 to 10 -------------------------
   if x > y then y : insert x ys
            else x : ys

sort :: [Char] -> [Char]
sort [] = []
sort (x:xs) = insert x ( sort xs )
```

Call Stack

```
Program terminated with error:
        No match in pattern.
Virtual stack trace:
(Last.hs:6)      last’ []
(Last.hs:6)      last’ [_]
(Last.hs:6)      last’ [_,_]
(Last.hs:4)      last’ [8,_,_]
(unknown)        main
```

Redex Trails

Redex Trails

```
Output: -----------------------------------------
os\n

Trail: ------- Insert.hs line: 10 col: 25 ------------------
<- putStrLn "os"
<- insert 's' "o" | if True
<- insert 'o' "r" | if False
<- insert 'r' "t" | if False
<- insert 't' []
<- sort []
```

Redex Trails

- Colin Runciman and Jan Sparud, 1997.
- Go backwards from observed failure to fault.
- Which redex created this expression?
- Based on graph rewriting semantics of abstract machine.

Animation of Lazy Evaluation

```
Output: ------------------------------------------------------

Animation: ---------------------------------------------------
-> sort "sort"
-> insert 's' ( sort "ort" )
-> insert 's' ( insert 'o' ( sort "rt" ) )
-> insert 's' ( insert 'o' ( insert 'r' ( sort "t" ) ) )
-> insert 's' ( insert 'o' ( insert 'r' "t" ) )
-> "os"
```

Trusting

Trust a module: Do not trace functions in module.

- Smaller trace file.
- Avoid viewing distracting details.

  `4 + 7 = 11`

Trusting

Trust a module: Do not trace functions in module.

- Smaller trace file.
- Avoid viewing distracting details.
  `4 + 7 = 11`

A trusted function may call a non-trusted function:

```
map prime [2,3,4,5] = [True,True,False,True]
```

Trusting

Trust a module: Do not trace functions in module.

- Smaller trace file.
- Avoid viewing distracting details.
  `4 + 7 = 11`

A trusted function may call a non-trusted function:

```
map prime [2,3,4,5] = [True,True,False,True]
```

In future?

- View-time trusting.
- Trusting of local definitions.

New Views

New Ideas

- Follow a value through computation.

New Views

New Ideas

- Follow a value through computation.

Combining Existing Views

- Can easily switch from one view to another.
- All-in-one tool = egg-laying wool-milk-sow?
- Exploring combination of algorithmic debugging and redex trails.

New Views

New Ideas

- Follow a value through computation.

Combining Existing Views

- Can easily switch from one view to another.
- All-in-one tool = egg-laying wool-milk-sow?
- Exploring combination of algorithmic debugging and redex trails.

Refining Existing Views

Algorithmic Debugging:

- Different Tree-Traversal Strategies.
- Heuristics.

Why a Theory of Tracing?

- Implementations of tracing tools ahead of theoretical results.
- Correctness of tools?
- Clear methodology for using them?
- Development of advanced features?

What is a Good Trace?

Program + input determine every detail of computation.

What is a Good Trace?

Program + input determine every detail of computation.
$\Rightarrow$ Trace gives efficient access to certain details of computation.

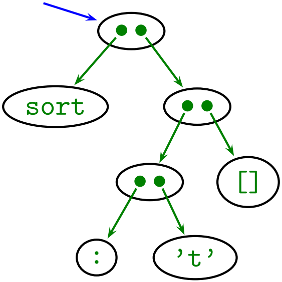
What is a Good Trace?

Program + input determine every detail of computation.
$\Rightarrow$ Trace gives efficient access to certain details of computation.

What is a computation? Semantics answers:

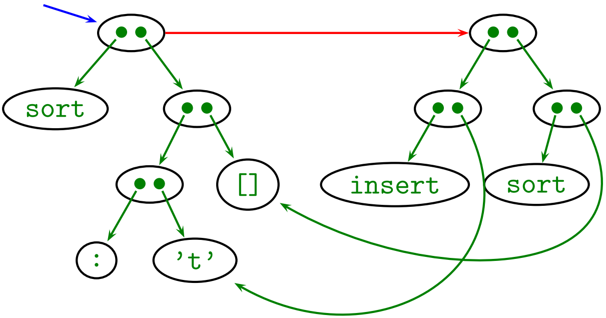
- Term rewriting: A sequence of expressions.
  $t_1 \longrightarrow t_2 \longrightarrow t_3 \longrightarrow t_4 \longrightarrow t_5 \longrightarrow \ldots \longrightarrow t_n$
- Natural semantics: A proof tree.

The Trace: Simple Graph Rewriting



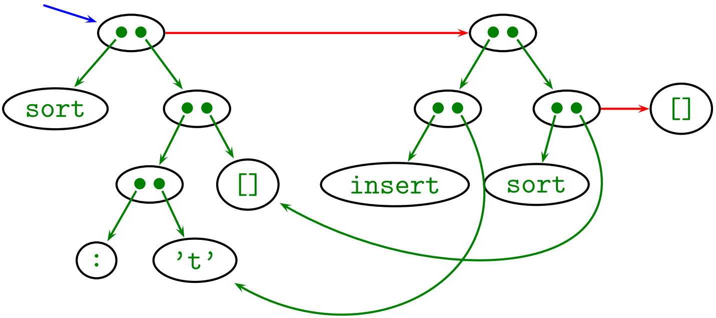
Start with expression `sort ('t':[])`

The Trace: Simple Graph Rewriting



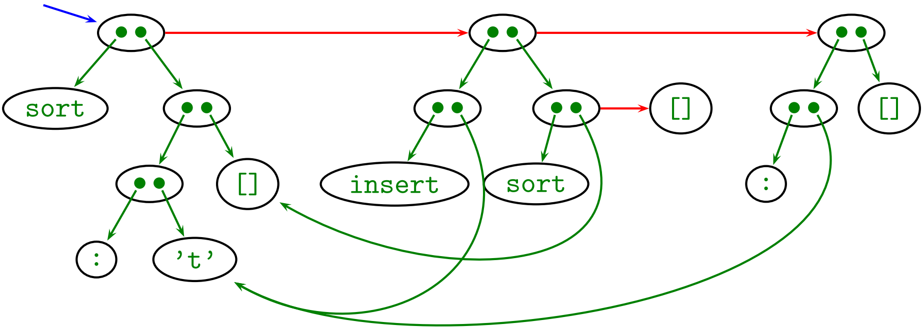

```
sort [] = []
sort (x:xs) = insert x (sort xs)
insert x [] = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x : ys
```

The Trace: Simple Graph Rewriting



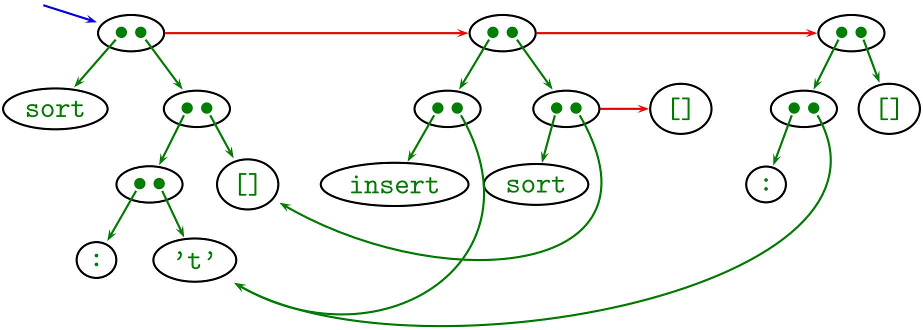

```
sort [] = []
sort (x:xs) = insert x (sort xs)

insert x [] = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x : ys
```

The Trace: Simple Graph Rewriting



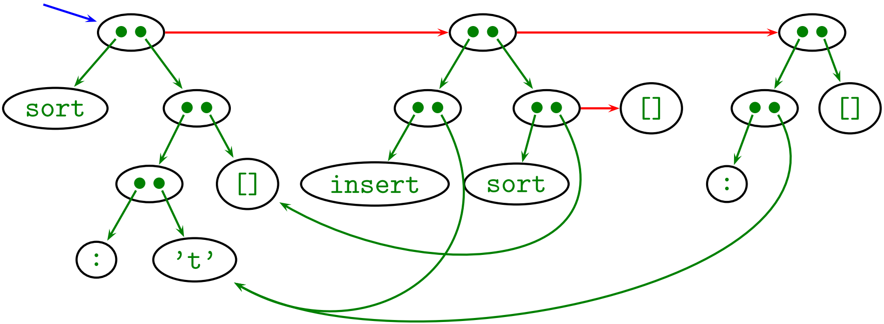

```
sort [] = []
sort (x:xs) = insert x (sort xs)
insert x [] = [x]
insert x (y:ys) = if x > y then y : (insert x ys) else x : ys
```

The Trace: Simple Graph Rewriting



- New nodes for right-hand-side, connected via result pointer.
- Only add to graph, never remove.
- Sharing ensures compact representation.

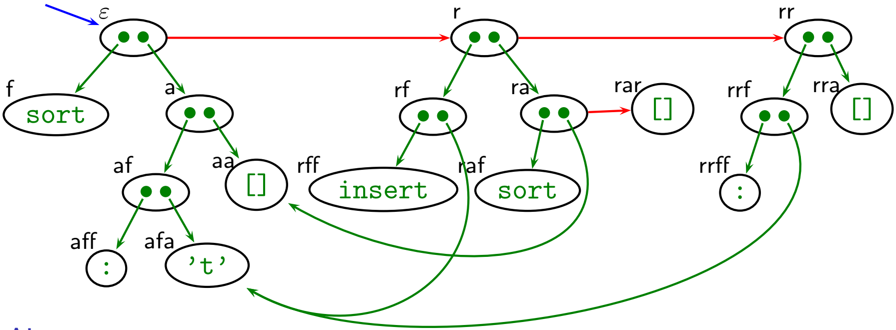
The Node Naming Scheme



Aim

- not distinguish isomorphic graphs
- avoid inconvenience of isomorphism classes

The Node Naming Scheme



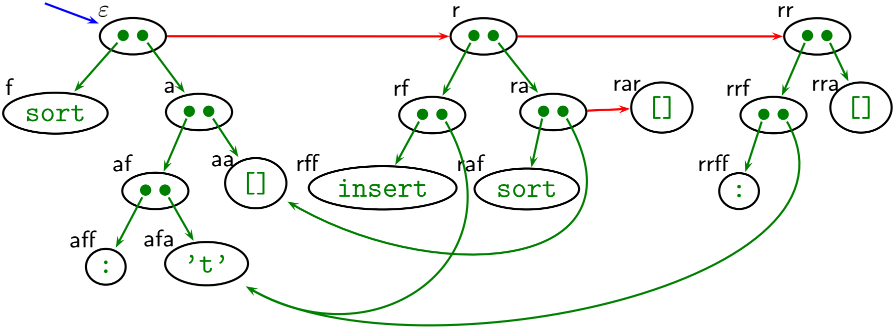
Aim

- not distinguish isomorphic graphs
- avoid inconvenience of isomorphism classes

Solution

- standard representation with node describing path from root
- path at creation time (sharing later)
- path independent of evaluation order

The Node Labels



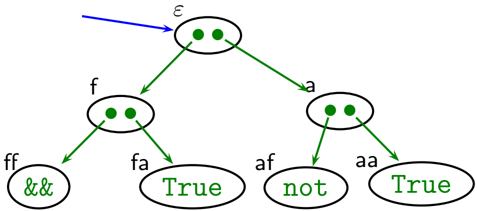
node	n	:=	$\{\mathsf{f}, \mathsf{a}, \mathsf{r}\}^*$	
label term	T	:=	a	atom
		|	$n\ m$	application of nodes
atom	a	:=	$f \mid C \mid 42 \mid \ldots$	defined variable, data constructor atomic literal, ...

Reduction edge implicitly given through existence of node.

Projections

- Reduction edge implicitly given through existence of node.
- Every redex should be parent of at least one node.
  (otherwise reduction unreachable from computation result)

```
True && x = x
not True = False
```

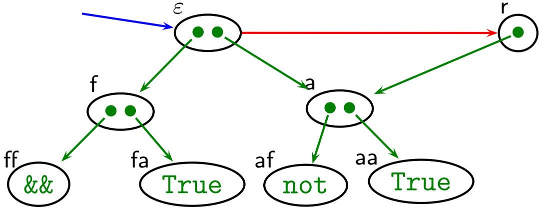


Projections

- Reduction edge implicitly given through existence of node.
- Every redex should be parent of at least one node.
  (otherwise reduction unreachable from computation result)

$\Rightarrow$ A projection requires an indirection as result.

```
True && x = x
not True = False
```



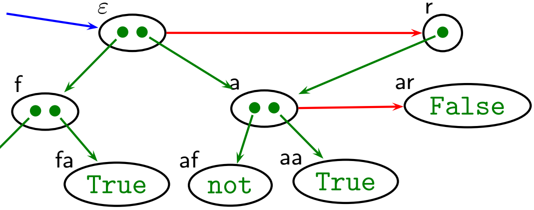
label term	T	$:=$	a	atom
		$\mid$	$n\ m$	application of nodes
		$\mid$	n	indirection
atom	a	$:=$	$x \mid C \mid 42 \mid \ldots$	variable, data constructor, ...

Projections

- Reduction edge implicitly given through existence of node.
- Every redex should be parent of at least one node.
  (otherwise reduction unreachable from computation result)

⇒ A projection requires an indirection as result.

```
True && x = x
not True = False
```



label term	T	$:=$	a	atom
		$\mid$	$n\ m$	application of nodes
		$\mid$	n	indirection

atom	a	$:=$	$x \mid C \mid 42 \mid \ldots$	variable, data constructor, ...

The Trace: The Augmented Redex Trail (ART)

A trace $\mathcal{G}$ for initial term M and program P is a partial function from nodes to term constructors, $\mathcal{G} : n \mapsto T$, defined by

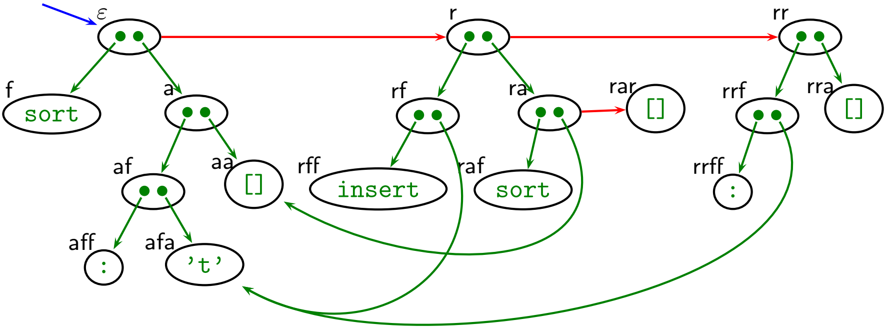
- The unshared graph representation of M, $\mathrm{graph}_{\mathcal{G}}(\varepsilon, M)$, is a trace.
- If $\mathcal{G}$ is a trace and
  - $L = R$ an equation of the program P,
  - σ a substitution replacing argument variables by nodes,
  - $\mathrm{match}_{\mathcal{G}}(n, L\sigma)$,
  - $n\mathrm{r} \notin \mathrm{dom}(\mathcal{G})$,

  then $\mathcal{G} \cup \mathrm{graph}_{\mathcal{G}}(n\mathrm{r}, R\sigma)$ is a trace.

No evaluation order is fixed.

The Most Evaluated Form of a Node

A node represents many terms, in particular a most evaluated one.



$\mathsf{mef}_{\mathcal{G}}(\varepsilon) = \texttt{(:) 't' []}$

Definition

$$n \succ_{\mathcal{G}} m \quad \Leftrightarrow \quad m = n\mathsf{r} \vee \mathcal{G}(n) = m$$

$$\lceil n \rceil_{\mathcal{G}} = m \quad \Leftrightarrow \quad n \succ^{*}_{\mathcal{G}} m \wedge \nexists o.\, m \succ_{\mathcal{G}} o$$

Definition

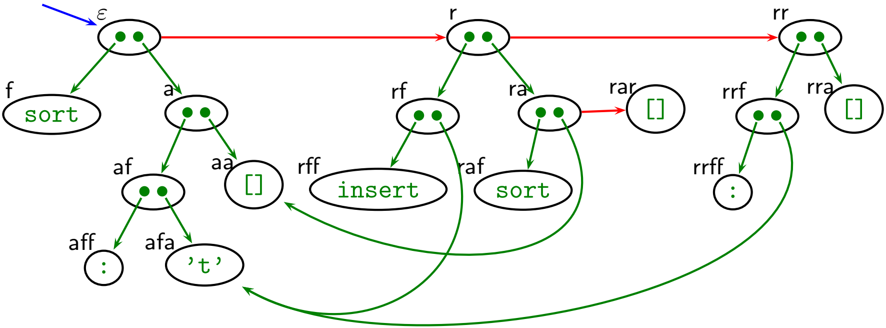
$$\mathsf{mef}_{\mathcal{G}}(n) = \mathsf{mefT}_{\mathcal{G}}(\mathcal{G}(\lceil n \rceil_{\mathcal{G}}))$$

$$\mathsf{mefT}_{\mathcal{G}}(a) = a$$

$$\mathsf{mefT}_{\mathcal{G}}(n) = \mathsf{mef}_{\mathcal{G}}(n)$$

$$\mathsf{mefT}_{\mathcal{G}}(n\, m) = \mathsf{mef}_{\mathcal{G}}(n)\ \mathsf{mef}_{\mathcal{G}}(m)$$

Redexes and Big-Step Reductions



$\text{redex}_{\mathcal{G}}(\text{r}) = $ `insert 't' []`

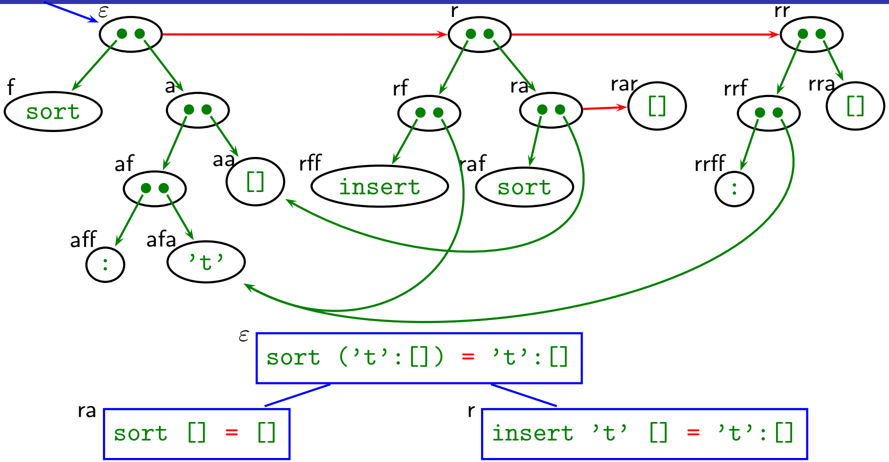
$\text{bigstep}_{\mathcal{G}}(\text{r}) = $ `insert 't' [] = (:) 't' []`

Definition

For any redex node n, i.e., $nr \in \text{dom}(G)$,

$$\text{redex}_{\mathcal{G}}(n) = \begin{cases} \text{mef}_{\mathcal{G}}(m)\ \text{mef}_{\mathcal{G}}(o) & \text{, if } \mathcal{G}(n) = m\, o \\ a & \text{, if } \mathcal{G}(n) = a \end{cases}$$

$$\text{bigstep}_{\mathcal{G}}(n) = \text{redex}_{\mathcal{G}}(n) = \text{mef}_{\mathcal{G}}(n)$$

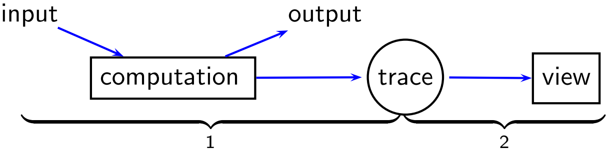


- Every redex node n yields a tree node n labelled $\mathrm{bigstep}_{\mathcal{G}}(n)$.
- Tree node n is child of tree node $\mathrm{parent}(n)$.

$$\mathrm{parent}(nr) = n$$
$$\mathrm{parent}(nf) = \mathrm{parent}(n)$$
$$\mathrm{parent}(na) = \mathrm{parent}(n)$$
$$\mathrm{parent}(\varepsilon) = \mathrm{undefined}$$

Summary

- Two-Phase Tracing.



  Liberates from time arrow of computation.
- There exist many useful different views of a computation.
  - Observation of Functions
  - Algorithmic Debugging
  - Source-based Free Navigation
  - Redex Trails

  ...
- Semantics.
  - Inspire views.
  - Enable formulation and proof of properties.
  - But do not answer all questions.
- Still much to explore.