Observational Advantages – Analysis By Task Type

Monday 21 October, 2019

Methods and Software

Statistical analysis was based on the Generalized Estimating Equations method (Liang and Zeger 1986) as this is implemented in the R (R Core Team 2019) package geepack (Højsgaard, Halekoh, and Yan 2005). In addition, the functions `ComparisonStats` and `AllContrasts` were developed to evaluate the statistical significance of the desired comparisons for the accuracy and time data.

```
> ComparisonStats <- function(FittedModel, Lmatrix, alpha = 0.05) {
+     Lmatrix <- matrix(Lmatrix, nrow = 1)
+     ModelBetas <- FittedModel$geese$beta
+     ModelVCov <- FittedModel$geese$vbeta
+     Estimate <- drop(Lmatrix %*% ModelBetas)
+     SdError <- sqrt(drop(Lmatrix %*% ModelVCov %*% t(Lmatrix)))
+     CBs <- Estimate + qnorm(c(alpha/2, 1 - alpha/2)) * SdError
+     pvalue <- 2 * pnorm(-abs(Estimate/SdError))
+     ans <- c(exp(c(Estimate, CBs)), round(pvalue, 4))
+     names(ans) <- c("Estimate", paste0((1 - alpha) * 100, "% LB"), paste0((1 -
+         alpha) * 100, "% UB"), "p-value")
+     ans
+ }
```

Import Data

The full data were imported by executing the following commands:

```
> library(readxl)
> free_rides <- read_excel("data/AllMainStudy_Data.xlsx", col_types = c("numeric",
+     "skip", "text", "text", "numeric", "text", "numeric", "numeric", "numeric",
+     "numeric", "text", "text", "text", "numeric", "text", "text", "numeric"))
> names(free_rides) <- gsub("[.]", "_", make.names(names(free_rides), unique = TRUE))
> free_rides <- free_rides[free_rides$Study_Id != 392, ]
```

Analysis of accuracy data

The following regression model was fitted to the accuracy data

$$\log\left[\frac{\Pr(Y_{ij}=1)}{1-\Pr(Y_{ij}=1)}\right] = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i1} x_{i4} + \beta_6 x_{i1} x_{i4} + \beta_7 x_{i3} x_{i4}$$

where

- $\Pr(Y_{ij}=1)$ is the probability for participant i to answer question j correctly.
- x_{i1} is the indicator for the *Linear* Treatment,
- x_{i2} is the indicator for the *Text* Treatment,
- x_{i3} is the indicator for the *Venn* Treatment,

- x_{i4} is the indicator for the *subset* Question Type,

for $i = 1, \ldots, 418$, corresponding to the individual participants, and $j = 1, \ldots, 20$ corresponding to the questions (questions $6 - 21$ are labelled as $5, \ldots, 20$) respectively.

```
> library(geepack)
> full_model <- geeglm(formula = Correct ~ Treatment * Question_Type, id = Study_Id,
+     data = free_rides, family = binomial)
```

Comparing this model to the model with no interaction term

```
> reduced_model <- geeglm(formula = Correct ~ factor(Treatment) + factor(Question_Type),
+     id = Study_Id, data = free_rides, family = binomial)
> Waldts <- anova(full_model, reduced_model)
> Waldts
Analysis of 'Wald statistic' Table

Model 1 Correct ~ Treatment * Question_Type
Model 2 Correct ~ factor(Treatment) + factor(Question_Type)
  Df     X2 P(>|Chi|)
1  3 77.228 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

we can conclude that at least one of the interplay of *Treatment* and *Question Type* is significant (p - value $= 1.110223 \times 10^{-16}$).

Comparison of treatments for the same question type

For $Question\_Type = disjoint$: $Euler = Linear > Venn > Text$.

For $Question\_Type = subset$: $Euler = Linear > Venn = Text$.

Question Type: disjoint

```
> ## (Linear & disjoint) versus (Text & disjoint)
> ComparisonStats(full_model, c(0, 1, -1, 0, 0, 0, 0, 0))
 Estimate     95% LB     95% UB    p-value
12.055816  7.888005 18.425787  0.000000
> ## (Linear & disjoint) versus (Venn & disjoint)
> ComparisonStats(full_model, c(0, 1, 0, -1, 0, 0, 0, 0))
Estimate   95% LB   95% UB  p-value
4.669429 2.963355 7.357731 0.000000
> ## (Linear & disjoint) versus (Euler & disjoint)
> ComparisonStats(full_model, c(0, 1, 0, 0, 0, 0, 0, 0))
 Estimate     95% LB     95% UB    p-value
0.6997571 0.4072662 1.2023096 0.1961000
> ## (Text & disjoint) versus (Venn & disjoint)
> ComparisonStats(full_model, c(0, 0, 1, -1, 0, 0, 0, 0))
 Estimate     95% LB     95% UB    p-value
0.3873176 0.2815730 0.5327744 0.0000000
> ## (Text & disjoint) versus (Euler & disjoint)
> ComparisonStats(full_model, c(0, 0, 1, 0, 0, 0, 0, 0))
  Estimate      95% LB      95% UB     p-value
0.05804312 0.03762788 0.08953477 0.00000000
```

```
> ## (Venn & disjoint) versus (Euler & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 1, 0, 0, 0, 0))
  Estimate     95% LB     95% UB    p-value
0.14985925 0.09428788 0.23818326 0.00000000
```

Question Type: subset

```
> ## (Linear & subset) versus (Text & subset)
> ComparisonStats(full_model, c(0, 1, -1, 0, 0, 1, -1, 0))
Estimate   95% LB   95% UB  p-value
5.148939 3.311551 8.005789 0.000000
> ## (Linear & subset) versus (Venn & subset)
> ComparisonStats(full_model, c(0, 1, 0, -1, 0, 1, 0, -1))
 Estimate    95% LB    95% UB   p-value
 6.814146  4.278204 10.853290  0.000000
> ## (Linear & subset) versus (Euler & subset)
> ComparisonStats(full_model, c(0, 1, 0, 0, 0, 1, 0, 0))
 Estimate    95% LB    95% UB   p-value
1.0454030 0.6227744 1.7548366 0.8666000
> ## (Text & subset) versus (Venn & subset)
> ComparisonStats(full_model, c(0, 0, 1, -1, 0, 0, 1, -1))
 Estimate    95% LB    95% UB   p-value
1.3234078 0.9440155 1.8552748 0.1040000
> ## (Text & subset) versus (Euler & subset)
> ComparisonStats(full_model, c(0, 0, 1, 0, 0, 0, 1, 0))
 Estimate    95% LB    95% UB   p-value
0.2030327 0.1351290 0.3050587 0.0000000
> ## (Venn & subset) versus (Euler & subset)
> ComparisonStats(full_model, c(0, 0, 0, 1, 0, 0, 0, 1))
  Estimate     95% LB     95% UB    p-value
0.15341657 0.09948609 0.23658226 0.00000000
```

Comparison of question types within treatment

For *Treatment* = *Euler* and *Venn*: *subset* < *disjoint*.

For *Treatment* = *Linear*: *subset* = *disjoint*.

For *Treatment* = *Text*: *subset* > *disjoint*.

```
> ## (Linear & subset) versus (Linear & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 0, 1, 1, 0, 0))
 Estimate    95% LB    95% UB   p-value
0.9897510 0.7022528 1.3949494 0.9531000
> ## (Text & subset) versus (Text & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 0, 1, 0, 1, 0))
Estimate   95% LB   95% UB  p-value
2.317420 1.878260 2.859262 0.000000
> ## (Venn & subset) versus (Venn & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 0, 1, 0, 0, 1))
 Estimate    95% LB    95% UB   p-value
0.6782320 0.5397152 0.8522990 0.0009000
> ## (Euler & subset) versus (Euler & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 0, 1, 0, 0, 0))
```

```
 Estimate    95% LB     95% UB    p-value
0.6625056 0.4934778 0.8894295 0.0062000
```

"‘

Analysis of time data

The following regression model was fitted to the time data

$$\begin{aligned}\log\left(Z_{ij}\right) =& \gamma_0 + \gamma_1 x_{i1} + \gamma_2 x_{i2} + \gamma_3 x_{i3} + \gamma_4 x_{i4} \\ &+ \gamma_5 x_{i1} x_{i4} + \gamma_6 x_{i1} x_{i4} + \gamma_7 x_{i3} x_{i4}\end{aligned}$$

where

- Z_{ij} is the time participant i needed to answer question j correctly.
- x_{i1} is the indicator for the *Linear* Treatment,
- x_{i2} is the indicator for the *Text* Treatment,
- x_{i3} is the indicator for the *Venn* Treatment,
- x_{i4} is the indicator for the *subset* Question Type,

for $i = 1, \ldots, 418$, corresponding to the individual participants, and $j = 1, \ldots, 20$ corresponding to the questions (questions $6 - 21$ are labelled as $5, \ldots, 20$) respectively.

```
> library(geepack)
> free_rides$log_time <- log(free_rides$Time)
> full_model <- geeglm(formula = log_time ~ Treatment * Question_Type, id = Study_Id,
+     data = free_rides[free_rides$Correct == 1, ])
```

Comparing this model to the model with no interaction term

```
> reduced_model <- geeglm(formula = log_time ~ Treatment + Question_Type, id = Study_Id,
+     data = free_rides[free_rides$Correct == 1, ])
> Waldts <- anova(reduced_model, full_model)
> Waldts
Analysis of 'Wald statistic' Table

Model 1 log_time ~ Treatment * Question_Type
Model 2 log_time ~ Treatment + Question_Type
  Df     X2 P(>|Chi|)
1  3 49.651 9.482e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

we can conclude that at least one of the interplay of *Treatment* and *Question Type* is significant (p - value $= 9.4818486 \times 10^{-11}$).

Comparison of treatments for the same question type

For $Question\_Type = disjoint$: $Euler = Linear > Venn > Text$.

For $Question\_Type = subset$: $Euler = Linear > Venn = Text$.

Question Type: disjoint

```
> ## (Linear & disjoint) versus (Text & disjoint)
> ComparisonStats(full_model, c(0, 1, -1, 0, 0, 0, 0, 0))
 Estimate    95% LB    95% UB   p-value
0.4269674 0.3709858 0.4913966 0.0000000
> ## (Linear & disjoint) versus (Venn & disjoint)
> ComparisonStats(full_model, c(0, 1, 0, -1, 0, 0, 0, 0))
 Estimate    95% LB    95% UB   p-value
0.6178601 0.5606032 0.6809648 0.0000000
> ## (Linear & disjoint) versus (Euler & disjoint)
> ComparisonStats(full_model, c(0, 1, 0, 0, 0, 0, 0, 0))
 Estimate    95% LB    95% UB   p-value
0.9300329 0.8436832 1.0252204 0.1446000
> ## (Text & disjoint) versus (Venn & disjoint)
> ComparisonStats(full_model, c(0, 0, 1, -1, 0, 0, 0, 0))
Estimate   95% LB   95% UB  p-value
1.447090 1.258549 1.663876 0.000000
> ## (Text & disjoint) versus (Euler & disjoint)
> ComparisonStats(full_model, c(0, 0, 1, 0, 0, 0, 0, 0))
Estimate   95% LB   95% UB  p-value
2.178230 1.894172 2.504886 0.000000
> ## (Venn & disjoint) versus (Euler & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 1, 0, 0, 0, 0))
Estimate   95% LB   95% UB  p-value
1.505248 1.367369 1.657031 0.000000
```

Question Type: subset

```
> ## (Linear & subset) versus (Text & subset)
> ComparisonStats(full_model, c(0, 1, -1, 0, 0, 1, -1, 0))
 Estimate    95% LB    95% UB   p-value
0.6131544 0.5514253 0.6817937 0.0000000
> ## (Linear & subset) versus (Venn & subset)
> ComparisonStats(full_model, c(0, 1, 0, -1, 0, 1, 0, -1))
 Estimate    95% LB    95% UB   p-value
0.6418238 0.5820063 0.7077893 0.0000000
> ## (Linear & subset) versus (Euler & subset)
> ComparisonStats(full_model, c(0, 1, 0, 0, 0, 1, 0, 0))
 Estimate    95% LB    95% UB   p-value
0.9320574 0.8446969 1.0284529 0.1611000
> ## (Text & subset) versus (Venn & subset)
> ComparisonStats(full_model, c(0, 0, 1, -1, 0, 0, 1, -1))
 Estimate    95% LB    95% UB   p-value
1.0467573 0.9370003 1.1693707 0.4188000
> ## (Text & subset) versus (Euler & subset)
> ComparisonStats(full_model, c(0, 0, 1, 0, 0, 0, 1, 0))
Estimate   95% LB   95% UB  p-value
1.520102 1.360011 1.699038 0.000000
> ## (Venn & subset) versus (Euler & subset)
> ComparisonStats(full_model, c(0, 0, 0, 1, 0, 0, 0, 1))
Estimate   95% LB   95% UB  p-value
1.452201 1.309517 1.610432 0.000000
```

Comparison of question types within treatment

For *Linear*, *Venn* and *Euler*: *subset* = *disjoint*.

For *Text*: *disjoint* < *subset*.

```
> ## (Linear & subset) versus (Linear & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 0, 1, 1, 0, 0))
 Estimate    95% LB    95% UB   p-value
1.0334633 0.9963851 1.0719212 0.0774000
> ## (Text & subset) versus (Text & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 0, 1, 0, 1, 0))
 Estimate    95% LB    95% UB   p-value
0.7196476 0.6530738 0.7930078 0.0000000
> ## (Venn & subset) versus (Venn & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 0, 1, 0, 0, 1))
 Estimate    95% LB    95% UB   p-value
0.9948769 0.9363642 1.0570460 0.8681000
> ## (Euler & subset) versus (Euler & disjoint)
> ComparisonStats(full_model, c(0, 0, 0, 0, 1, 0, 0, 0))
 Estimate    95% LB    95% UB   p-value
1.0312186 0.9934368 1.0704373 0.1065000
```

References

Højsgaard, S., U. Halekoh, and J. Yan. 2005. "The R Package geepack for Generalized Estimating Equations." *Journal of Statistical Software* 15: 1–11.

Liang, K.Y., and S.L. Zeger. 1986. "Longitudinal Data Analysis Using Generalized Linear Models." *Biometrika* 73 (1): 13–22.

R Core Team. 2019. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. http://www.R-project.org/.