```
! subroutine Compute_Solution_* ( T, A, Z, N0, N_T, B )

    real(rk), intent(in) :: T          ! Independent variable
    real(rk), intent(in) :: A(:,:)     ! Coefficient matrix for ODE
    real(rk), intent(in) :: Z(:,:)     ! Coefficient matrix solutions
    real(rk), intent(in) :: N0(:)      ! Initial condition
    real(rk), intent(out) :: N_T(:)    ! Solution N(T)
    real(rk), intent(in), optional :: B(:)  ! For inhomogeneous equations.
      ! Let M be the index in A of the last nonzero diagonal element.
      ! Then B(1:m) is B in    d N(t)/d t = A ( N(t) + B )
      ! and B(m+1:) is C in    d N(t)/d t = A N(t) + C. Remember that here,
      ! columns of A after M are zero, so this is just a quadrature.
      ! Backsolve_A_C_B can compute B(1:m) if all you have is C.
    real(rk) :: E
    integer :: I, J, M, R, U

    u = ubound(a,1)

    ! Find the first zero diagonal in A.
    ! Assume A(i,j) = 0 for i>j if A(j,j) = 0.
    do m = 0, u - 1
      if ( a(m+1,m+1) == 0 ) exit
    end do

    if ( .not. present(b) ) then
```

Compute the homogeneous solution for the first m equations.

$$N_i(t) = N_i(0)\, e^{a_{ii} t} + \sum_{j=1}^{i-1} z_{ij} \left(N_j(t) - N_j(0)\, e^{a_{ii} t} \right) \quad i \leq m \tag{1}$$

```
      do i = 1, m
        e = exp(a(i,i) * t)
        N_T(i) = n0(i) * e + &
          & sum ( [ ( z(i,j) * ( N_T(j) - n0(j) * e ), j = 1, i-1 ) ] )
      end do

      if ( m < u ) then
```

Compute the integrals of the first m equations:

$$\int_0^t N_l(t)\,\mathrm{d}t = N_l(0)\,\frac{e^{a_{ll} t} - 1}{a_{ll}} + \sum_{j=1}^{l-1} z_{lj} \left(\int_0^t N_j(t)\,\mathrm{d}t - N_j(0)\,\frac{e^{a_{ll} t} - 1}{a_{ll}} \right),\ l \leq m\,. \tag{2}$$

```

        block
```

```
          real(rk) :: N_Int(m)
          do i = 1, m
            e = ( exp(a(i,i) * t) - 1 ) / a(i,i)
            n_int(i) = n0(i) * e + &
              & sum( [ ( z(i,j) * ( n_int(j) - n0(j) * e ), j = 1, i-1 ) ] )
          end do
```

Compute the homogeneous solution for the remaining $m+1:u$ equations:

$$N_i(t) = \sum_{l=1}^{m} a_{il} \int_0^t N_l(t)\,\mathrm{d}t + N_i(0)\,,\; i > m \tag{3}$$

```

          n_t(m+1:u) = matmul(a(m+1:u,1:m), n_int(1:m))+ n0(m+1:u)
        end block
      end if
    else
```

Compute the inhomogeneous solution for the leading m equations,

$$N_i(t) = \hat{N}_i(0)\, e^{a_{ii}t} + \sum_{j=1}^{i-1} z_{ij} \left(\hat{N}_j(t) - \hat{N}_j(0)\, e^{a_{ii}t} \right) - B_i\,,\; i \le m\,, \tag{4}$$

where $\hat{\mathbf{N}}(0) = \mathbf{N}(0) + \mathbf{B}$.

```
      do i = 1, m
        e = exp(a(i,i)*t)
        N_T(i) = ( n0(i) + b(i) ) * e - b(i) + &
          & sum ( [ ( z(i,j) * ( N_T(j) + b(j) - ( n0(j) + b(j) ) * e ), &
                    & j = 1, i-1 ) ] )
      end do

      if ( m < u ) then
```

Compute the integrals of the first m equations:

$$\int_0^t N_l(t)\,\mathrm{d}t = \hat{N}_l(0)\, \frac{e^{a_{ll}t}-1}{a_{ll}} + \sum_{j=1}^{l-1} z_{lj} \left(\int_0^t \hat{N}_j(t)\,\mathrm{d}t - \hat{N}_j(0)\, \frac{e^{a_{ll}t}-1}{a_{ll}} \right) - B_i\, t\,,\; l \le m\,, \tag{5}$$

where $\hat{\mathbf{N}}(0) = \mathbf{N}(0) + \mathbf{B}$.

```
        block
          real(rk) :: N_Int(size(n_t))
          do i = 1, m
            e = ( exp ( a(i,i) * t ) - 1 ) / a(i,i)
            n_int(i) = ( n0(i) + b(i) ) * e - b(i) * t + &
              & sum ( [ ( z(i,j) * ( n_int(j) + b(j)*t - &
                        & ( n0(j) + b(j) ) * e ), j = 1, i-1 ) ] )
          end do
```

Compute the inhomogeneous solution for the remaing $n - m$ equations:

$$N_i(t) = \sum_{l=1}^{m} a_{il} \int_0^t N_l(t)\,\mathrm{d}t + C_i t + N_i(0)\,, i > m\,. \tag{6}$$

Here, C_i is in the array `B`.

```
          n_t(m+1:u) = matmul(a(m+1:u,1:m), n_int(1:m)) + b(m+1:u)*t + n0(m+1:u)
        end block
      end if
    end if

! end subroutine Compute_Solution_*
```